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Solve each problem using the four-step plan.

1. **Engineering** Geothermal energy is heat from inside the earth. Underground temperatures usually rise 9°C for each 300 feet of depth. For the ground temperature to rise 90°C, how deep would you have to dig?

2. **Transportation** A DC-11 jumbo jet carries 342 passengers with 36 in first class seating and the rest in coach. On a certain day, a first class ticket from Los Angeles to Chicago costs $750, and a coach ticket costs $450. What will be the ticket sales for the airline if the flight is full?

3. **Retail** At the Woodward Park School bookstore, a ball point pen costs 28¢ and a notepad costs 23¢. What could you buy and spend exactly 74¢?

4. **Geography** On Kenny’s map, each inch represents 30 miles. Raleigh, NC, is about 12 inches from Atlanta, GA. About how many miles is this?

5. **Fundraising** Emily sold 332 boxes of candy in two weeks for her band trip. If she sold the same number of boxes each day, how many boxes of candy did Emily sell each day?

6. **Catering** Swan Catering Services offers banquet facilities and food service. They charge $6 per person for a cold buffet. If the Everettts invite 75 people to a retirement party, how much should they budget for the cold buffet?
There are rules for the order of operations.

1. Do all operations within grouping symbols first; start with the innermost grouping symbols.
2. Next, do all multiplications and divisions from left to right.
3. Then, do all additions and subtractions from left to right.

3 + 5 \times 7 \quad \text{First multiply.} \quad 16 \div 2 \times 3 \quad \text{From left to right, divide.}
= 3 + 35 \quad \text{Then add.} \quad 8 \times 3 \quad \text{Then multiply.}
= 38

**Name the operation that should be done first. Then find the value.**

1. 24 \div 3 + 6
2. 6 \times 9 - 7
3. 14 \div 2 \times 7
4. 13 + 28 \div 4 + 5
5. (22 + 6) \times 8
6. 16 \div (8 - 4)

**Find the value of each expression.**

7. \( \frac{25}{5} - 2 \)
8. \( \frac{7 + 3}{10} - 5 \)
9. \( \frac{7 \times 9 + 6}{63 + 6} \)

10. \( \frac{(12 - 8)}{4} + 6 \)
11. \( 6 \times 8 \div 12 + 3 \)
12. \( 5 + 8 \times 2 - 7 \)
13. \( 24 + 6 \times 2 \)
14. \( 12 \times 5 \div 4 \)
15. \( 13 + 5 \times 4 \)
16. \( 50 - 28 \div 4 \times 7 \)
17. \( 36 - 3 \times 2 \times 4 \)
18. \( 60 \div 12 \times (4 - 1) \)
19. \( (100 - 25) \times 2 + 25 \)
20. \( 3 \times 7 - 5 + 4 \)
21. \( 9 \times 4 \div 2 - 10 \)
A variable is used to represent some number. The value of an expression may be changed by replacing a variable with different numbers. An expression may contain more than one variable. Remember to use the order of operations when evaluating expressions.

\textbf{Example:} Evaluate $5a - 2b$ if $a = 3$ and $b = 2$.  
\begin{align*}
5a - 2b &= 5(3) - 2(2) \\
&= 15 - 4 \\
&= 11
\end{align*}

\textbf{Example:} Evaluate $5a - 2b$ if $a = 10$ and $b = 0$.  
\begin{align*}
5a - 2b &= 5(10) - 2(0) \\
&= 50 - 0 \\
&= 50
\end{align*}

\textbf{Evaluate each expression if $a = 1$, $b = 2$, $x = 5$, and $y = 10$.}

1. $y - a$  
2. $y - b$  
3. $x + y$  
4. $a + b$

5. $ab$  
6. $ax$  
7. $bx$  
8. $xy$

9. $ab + y$  
10. $ax - b$  
11. $bx - y$  
12. $xy + ab$

13. $\frac{y}{b}$  
14. $\frac{y}{x}$  
15. $\frac{x}{a} + x$  
16. $\frac{y}{a} - bx$

17. $5a$  
18. $5b$  
19. $10y + y$  
20. $10b - a$

\textbf{Evaluate each expression if $a = 3$, $b = 5$, $x = 2$, and $y = 4$.}

21. $a \times 7$  
22. $b \times 8$  
23. $2 \times x + y$

24. $a + 6 \times b$  
25. $5x - y$  
26. $2b - 2a$

27. $6x \div y$  
28. $x + y \times 7 \div 2$  
29. $a \times x + b \times y$

30. $2ab$  
31. $b + x \times a$  
32. $4x \div y$
Three Properties of Addition | Four Properties of Multiplication
---|---
**Commutative** | **Commutative**
The order of adding does not change the sum.
15 + 23 = 23 + 15 | The order of multiplying does not change the product.
3 × 5 = 5 × 3
**Associative** | **Associative**
Grouping numbers differently does not change the sum.
(18 + 2) + 8 = 18 + (2 + 8) | Grouping numbers differently does not change the product.
(3 × 2) × 4 = 3 × (2 × 4)
**Identity** | **Identity**
Adding zero to a number does not change its value.
68 + 0 = 68 | Multiplying a number by 1 does not change its value.
5 × 1 = 5
**Zero** | **Zero**
Zero times any number equals zero.
16 × 0 = 0

**Name the addition property shown by each statement.**

1. 300 + 0 = 300
2. 96 + 4 = 4 + 96
3. 0 + 36 = 36
4. (6 + 2) + 4 = 6 + (2 + 4)
5. 20 + (12 + 2) = (20 + 12) + 2
6. 27 + 45 = 45 + 27
7. 75 + 25 = 25 + 75
8. 2115 = 2115 + 0
9. (3 + 7) + 6 = 3 + (7 + 6)
10. x + y = y + x
11. y + 0 = y
12. x + (y + z) = (x + y) + z

**Name the multiplication property shown by each statement.**

13. 3 × 2 = 2 × 3
14. (6 × 3) × 4 = 6 × (3 × 4)
15. 7 × 1 = 7
16. 0 × 8 = 0
17. 76 × 13 = 13 × 76
18. 42 × 0 = 0
19. xy = yx
20. x(yz) = (xy)z
21. 1 × y = y
The distributive property involves multiplication and addition. It states that the sum of two products with a common factor is equal to the sum of the other factors times the common factor.

\[(12 \times 5) + (4 \times 5) = (12 + 4) \times 5\]

Notice that the two products on the left have a common factor, 5.

The distributive property is used to combine like terms in algebraic expressions. Two or more terms with the same variables are combined by adding the coefficients.

\[12a + 4a = (12 + 4)a = 16a\]
\[b + 6b = (1 + 6)b = 6b\]

**Use the distributive property to find the value of each of the following.**

1. \((3 \times 8) + (3 \times 2)\)
2. \((2 \times 9) + (2 \times 11)\)

3. \((13 \times 6) + (7 \times 6)\)
4. \((59 \times 5) + (41 \times 5)\)

5. \((25 \times 731) + (25 \times 129)\)
6. \((486 \times 40) + (144 \times 40)\)

7. \((6 \times 31) + (6 \times 147)\)
8. \((7 \times 4) + (7 \times 9) + (7 \times 8)\)

**Simplify each expression.**

9. \(8y + 2y\)
10. \(9m + 2m\)
11. \(13a + 7a\)

12. \(5x + x\)
13. \(d + 10d\)
14. \(z + z\)

15. \(8y + 2y + 3y\)
16. \(9m + 2m + 3m\)
17. \(13a + 7a + 2a\)

18. \(8y + 2y + y\)
19. \(m + 2m + m\)
20. \(a + a + 2a\)
The solution of an equation is the number or numbers that make it true. When you have found the solution of an equation, you have solved it.

One way to solve an equation is to guess a number and then check to see if your guess is correct.

**Example:** Solve \( m + 13 = 20 \)

First guess: 33  
\( 33 + 13 = 46 \)
So 33 is not the solution.

Next guess: 10  
\( 10 + 13 = 23 \)
So 10 is not the solution.  
But, 10 is closer than 33.

Next guess: 7  
\( 7 + 13 = 20 \)
So 7 is the solution to the equation.

**Name the number that is a solution of the given equation.**

1. \( 19 \times p = 95; 0, 1, 2, 3, 4, 5, 6 \)  
2. \( 11 - q = 8; 1, 3, 5, 7 \)

3. \( 51 \div r = 17; 0, 3, 6, 9, 12 \)  
4. \( 21 + j = 31; 0, 5, 10, 15, 20 \)

5. \( 28 = 4 \times s; 1, 3, 5, 7, 9 \)  
6. \( 13 - t = 9; 0, 2, 4, 6, 8, 10 \)

7. \( 100 = h + 25; 0, 25, 50, 75 \)  
8. \( g \times 9 = 0; 0, 3, 6, 9, 12 \)

**Solve each equation mentally.**

9. \( m + 21 = 35 \)  
10. \( 17 - c = 2 \)  
11. \( 150 = 15 \times s \)

12. \( 9 = a \div 2 \)  
13. \( h + 23 = 32 \)  
14. \( c - 12 = 50 \)

15. \( 81 = 9 \times t \)  
16. \( 3 = 12 \div t \)  
17. \( 82 + a = 102 \)

18. \( 11 = h - 7 \)  
19. \( 6 \times a = 48 \)  
20. \( u \div 4 = 20 \)
An ordered pair of numbers, such as the point (5, 2), can be graphed as follows.

Move right 5 units.

Then move up 2 units.

**Describe the location of each point with respect to the point (0, 0).**

1. (4, 4)  
2. (1, 4)  
3. (6, 5)  
4. (5, 3)  
5. (0, 2)  
6. (2, 1)  
7. (3, 0)  
8. (2, 6)  
9. (3, 3)

**Use the grid at the right to name the point for each ordered pair.**

10. (4, 4)  
11. (5, 3)  
12. (3, 0)  
13. (1, 6)  
14. (2, 2)  
15. (3, 4)

**Use the grid at the right to find the ordered pair for each labeled point.**

16. A  
17. G  
18. B  
19. H  
20. C  
21. I  
22. D  
23. J
Addition and subtraction are inverse operations. Because of this, these four equations give the same information.

\[ 10 + 5 = 15 \quad 5 + 10 = 15 \quad 15 - 5 = 10 \quad 15 - 10 = 5 \]

Multiplication and division are also inverse operations.

\[ 4 \times 5 = 20 \quad 5 \times 4 = 20 \quad 20 \div 5 = 4 \quad 20 \div 4 = 5 \]

To solve a single-operation equation, write and solve the related equation that has the variable all by itself on one side of the equal sign.

\[ x + 5 = 15 \rightarrow 15 - 5 = x \quad 4 \times n = 20 \rightarrow 20 \div 4 = n \]

Write the three related equations for each given equation.

1. \(7 \times 6 = 42\)  
2. \(50 - 30 = 20\)

3. \(56 \div 8 = 7\)  
4. \(13 + 3 = 16\)

5. \(6 + m = 10\)  
6. \(12 = 3y\)

7. \(d = 32 \div 8\)  
8. \(12 = t - 3\)

Solve each equation by using the inverse operation. Use a calculator where necessary.

9. \(6x = 42\)  
10. \(42 = 7y\)  
11. \(42 \div z = 6\)

12. \(13 + a = 23\)  
13. \(b - 13 = 23\)  
14. \(23 - c = 13\)

15. \(56 - p = 0\)  
16. \(56 + q = 56\)  
17. \(r - 0 = 56\)

18. \(30 = 10k\)  
19. \(30 \div h = 10\)  
20. \(j \times 3 = 30\)
An **inequality** is a number sentence which states that two expressions are *not* equal. Four symbols for inequality are $>$, $<$, $\geq$, and $\leq$. Notice that the point of the V-shaped symbol points toward the lesser expression.

\[
2 + 2 > 3 \quad \text{Two plus two is greater than 3.} \\
2 + 2 \geq 3 \quad \text{Two plus two is greater than or equal to 3.} \\
2 + 2 \geq 4 \quad \text{Two plus two is greater than or equal to 4.} \\
2 + 2 < 5 \quad \text{Two plus two is less than 5.} \\
2 + 2 \leq 5 \quad \text{Two plus two is less than or equal to 5.} \\
2 + 2 \leq 4 \quad \text{Two plus two is less than or equal to 4.}
\]

**Translate each statement into an algebraic inequality.**

1. $x$ is less than 10.  
   \[x < 10\]

2. 20 is greater than or equal to $y$.  
   \[20 \geq y\]

3. 14 is greater than $a$.  
   \[14 > a\]

4. $b$ is less than or equal to 8.  
   \[b \leq 8\]

5. 6 is less than the product of $f$ and 20.  
   \[6 < f \times 20\]

6. The sum of $t$ and 9 is greater than or equal to 36.  
   \[t + 9 \geq 36\]

7. 7 more than $w$ is less than or equal to 10.  
   \[7 + w \leq 10\]

8. 19 decreased by $p$ is greater than or equal to 2.  
   \[19 - p \geq 2\]

**Name the numbers that are a solutions of the given inequality.**

9. $r > 10; 5, 10, 15, 20$  
10. $t \geq 10; 5, 10, 15, 20$

11. $2 + n < 5; 0, 1, 2, 3, 4$  
12. $2 + m \leq 5; 0, 1, 2, 3, 4$

13. $6 + m \geq 10; 3, 4, 5, 6, 7$  
14. $6 + m \leq 10; 3, 4, 5, 6, 7$

15. $30 \leq 5d; 4, 5, 6, 7, 8$  
16. $30 \geq 5d; 4, 5, 6, 7, 8$
To make a frequency table:
   a. collect the data.
   b. tally the results.
   c. count the tallies.

1. Complete the frequency table for the number of birthdays per month.

2. Which four months had the greatest number of birthdays?

3. Compare the number of birthdays in the first six months to the number in the second six months.

4. Complete the frequency table for the number of books read.

5. How many people responded to this survey?

6. What percent chose mysteries? (Round to the nearest whole percent.)

7. What is the ratio of those who chose romances to those who chose sports?

8. Complete the frequency table for these scores.

   **Math Quiz Scores**

<table>
<thead>
<tr>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>82 78</td>
</tr>
<tr>
<td>91 67</td>
</tr>
<tr>
<td>72 75</td>
</tr>
<tr>
<td>81 68</td>
</tr>
<tr>
<td>90 75</td>
</tr>
<tr>
<td>83 72</td>
</tr>
<tr>
<td>77 71</td>
</tr>
<tr>
<td>65 55</td>
</tr>
<tr>
<td>93 73</td>
</tr>
<tr>
<td>76 81</td>
</tr>
<tr>
<td>90 88</td>
</tr>
<tr>
<td>78 75</td>
</tr>
<tr>
<td>60</td>
</tr>
</tbody>
</table>

   **Favorite books**

<table>
<thead>
<tr>
<th>Style</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science Fiction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mystery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Biography</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Romance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sports</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Historical Novel</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   **Range**

<table>
<thead>
<tr>
<th>Range</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>95–100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90–94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>85–89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80–84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75–79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70–74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>65–69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>below 65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Situations that involve growth or increase are usually represented by positive integers.

Situations that involve decline or decrease are usually represented by negative integers.

<table>
<thead>
<tr>
<th>Situations Represented by Positive Integers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit of $50 \rightarrow +50</td>
</tr>
<tr>
<td>Deposit of $400 \rightarrow +400</td>
</tr>
<tr>
<td>Increase of 20 \rightarrow +20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Situations Represented by Negative Integers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss of $30 \rightarrow -30</td>
</tr>
<tr>
<td>Withdrawal of $250 \rightarrow -250</td>
</tr>
<tr>
<td>Decrease of 40 \rightarrow -40</td>
</tr>
</tbody>
</table>

Write the integer that describes the situation.

1. loss of 8 yards
2. 4° rise in temperature
3. 50-foot drop in altitude
4. debt of $500
5. deposit of $70
6. gain of 10 pounds

Graph each set of numbers on the number line provided.

7. \{-4, -1, 3\}  

8. \{-2, 0, 5\}

Write the absolute value of each integer.

9. -3
10. 14
11. 20
12. -5

Write the two integers that have the given absolute value.

13. 6
14. 1
15. 15
16. 8
17. 3
18. 12
19. 30
20. 21

Simplify.

21. \(|4| - |-2|\)
22. \(|-8| + |-3|\)
23. \(|-15| - |6|\)
24. \(|-7| \cdot |-11|\)
25. \(|12| \cdot |-4|\)
26. \(|-36| \div |-9|\)

© Glencoe/McGraw-Hill
A horizontal number line and a vertical number line that meet at zero are shown in the figure at the right.

Such a **coordinate system** is used for maps and graphs, and has many uses in mathematics.

Point A in the figure is the point for the **ordered pair** (2, 4).

To find point A, start at 0 and move to the point for 2 on the horizontal number line. Then move **up** 4 units. To find point B, start at 0 and move to the point for −3 on the horizontal number line. Then move **down** 5 units.

**Use the coordinate system at the right to name the ordered pair for each point.**

1. A  2. B  3. C  

**Use the coordinate system at the right. Graph and label each point. Name the quadrant in which each point is located.**

10. K(5, 2)  11. L(−1, 3)  12. M(4, −4)  
13. N(−2, −6)  14. P(3, 0)  15. Q(0, −2)  
16. R(6, −2)  17. S(−5, 2)  18. T(1, 5)
The graphs of -4, -1, 3, and 5 are shown on the number line.

The following statements can be made about the numbers and their graphs.

-4 is graphed to the left of -1, so -4 < -1.
-1 is graphed to the left of 5, so -1 < 5.
3 is graphed to the right of -1, so 3 > -1.
5 is graphed to the right of -4, so 5 > -4.

Write >, <, or = in each □.

1. 4 □ -4
2. 8 □ 12
3. -7 □ -5

4. 2 □ -1
5. -8 □ -8
6. -4 □ 3

7. -3 □ -8
8. -11 □ -10
9. 6 □ -9

10. -12 □ 7
11. 9 □ -9
12. 5 □ -5

Order the numbers in each set from least to greatest.

13. {4, -4, 2, -1}
14. {-5, 0, -3, 1}

15. {-9, -6, -12, -7}
16. {-1, 3, -5, -7}

17. {-5, 5, -4, 4}
18. {2, -3, 0, 4}

Cross out one number in each group so that the rest of the numbers are in order. More than one answer is possible.

19. -3, -2, 0, -1, 3, 4
20. 5, 4, 3, 1, 0, -3, -2, -4

21. -6, -4, -2, 0, 4, 2, 6
22. 15, 10, 0, 5, -5, -10, -15

23. -1, -2, 0, 1, 2, 3, 4
24. 9, 7, 5, -5, -9, -7, -11
2-4 Study Guide
Adding Integers

To add integers with different signs, find the difference of their absolute values. The sum has the same sign as the addend with the greater absolute value.

\[
7 + (-2) = 5 \quad -7 + 2 = -5
\]

To add integers with the same sign, add their absolute values. The sum has the same sign as the addends.

\[
8 + 3 = 11 \quad -8 + (-3) = -11
\]

To add more than two integers, follow these three steps:
1. Add all the positive integers.
2. Add all the negative integers.
3. Add these two sums together.

\[
-3 + 2 + (-6) = 2 + (-3) + (-6)
\]

\[
= 2 + (-9)
\]

\[
= -7
\]

\[
-8x + 9x + (-3)x = [-8 + (-3)]x + 9x
\]

\[
= -11x + 9x
\]

\[
= -2x
\]

Add.
1. \(6 + 4\)  
2. \(1 + 4\)  
3. \(-3 + (-2)\)  
4. \(-6 + 4\)
5. \(-1 + (-5)\)  
6. \(2 + (-4)\)  
7. \(-2 + 2\)  
8. \(5 + (-3)\)

Solve each equation.
9. \(-2 + (-1) + 6 = x\)  
10. \(-5 + 3 + 3 = y\)  
11. \(4 + 3 + (-2) = d\)
12. \(r = 9 + (-4) + 3\)  
13. \(m = -3 + (-8) + 11\)  
14. \(c = 2 + (-7) + (-1)\)

Simplify each expression.
15. \(-4x + 7x + (-6)x\)  
16. \(8f + (-2) + (-5)f\)  
17. \(-7e + (-10)e + 7e\)
18. \(3y + 6y + (-10)y\)  
19. \(-10t + 9t + 3t\)  
20. \(5d + (-1)d + (-8)d\)
To subtract an integer, add its opposite.

\[15 - 20 = 15 + (-20) = -5\]

To subtract 20, add \(-20\).

\[5 - (-9) = 5 + 9 = 14\]

To subtract \(-9\), add 9.

Write an addition expression for each of the following.

1. \(9 - 16\)
2. \(12 - (-8)\)
3. \(-7 - (-7)\)

4. \(-3 - 18\)
5. \(\frac{4}{9} - \frac{4}{9}\)
6. \(-3.5 - (-4.7)\)

Simplify each expression.

7. \(-5x - 5x\)
8. \(7y - (-12y)\)
9. \(4z - 15z\)

10. \(-6ab - (-11ab)\)
11. \(-21rs - (-14rs)\)
12. \(17np - (-9np)\)

13. \(42d - (-18d)\)
14. \(-17w - (-36w)\)
15. \(36c - (-81c)\)

16. \(-83k - (-38k)\)
17. \(-56t - (-41t)\)
18. \(15xy - (-6xy)\)

19. \(45p - 63p\)
20. \(-35f - (-35f)\)
21. \(-53uv - 32uv\)

Solve each equation.

22. \(5 - 11 = n\)
23. \(x = 9 - (-2)\)
24. \(d = 11 - 3\)

25. \(20 - 15 = a\)
26. \(b = 15 - 20\)
27. \(-15 - 20 = c\)

28. \(p = 3 - 3\)
29. \(q = -3 - 3\)
30. \(-3 - (-3) = r\)

31. \(50 - (-25) = d\)
32. \(-75 - 50 = e\)
33. \(-25 - (-50) = f\)
**Example:** At Fairview High School the bell rings at 7:35, 8:22, 8:25, 9:12, and 9:15 each weekday morning. When do the next three bells ring?

<table>
<thead>
<tr>
<th>Bell</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>7:35</td>
<td>8:22</td>
<td>8:25</td>
<td>9:12</td>
<td>9:15</td>
</tr>
</tbody>
</table>

**Explore** The chart shows the times that the bell rings.

**Plan** Since bell schedules often follow patterns, look for a pattern. Once you find a pattern, you can determine the next three bells.

**Solve** Notice that each class period is forty-seven minutes long and there is three minutes between each class. According to this pattern, the next three bells will be at 10:02, 10:05, and 10:52.

**Examine** The first class begins at 7:35. The class periods are forty-seven minutes in length, so dismissal bell will ring at 8:22. Having three minutes between classes, you must be in the next class by 8:25. Continuing this pattern, you will see that the sixth, seventh, and eighth bells ring at 10:02, 10:05, and 10:52.

**Solve. Look for a pattern.**

1. Find the next two integers in the set. \{3, 5, 9, 17, 33, , , \}

2. Jon ran 1 lap on the first day. He ran 2 laps on the second day, 4 laps on the third day, and 8 laps on the fourth day. How many laps should he run on the seventh day?

3. The following chart shows the scores at the baseball game.

<table>
<thead>
<tr>
<th>Cubs</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>0</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reds</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>3</td>
<td>?</td>
</tr>
</tbody>
</table>

If the pattern continues, how many runs will the Reds score in the bottom of the 7th inning?

4. Ellie is using the following chart to help her calculate the discount on T-shirts.

<table>
<thead>
<tr>
<th>T-shirts</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount</td>
<td>$1.75</td>
<td>$3.25</td>
<td>$4.75</td>
</tr>
</tbody>
</table>

Mr. Smith would like to order 8 T-shirts. How much of a discount should Ellie give him?
The product of two integers with different signs is negative.

\[8 \times -2 = -16\] \quad \[8 \times 2 = -16\]

The product of two integers with same signs is positive.

\[5 \times 6 = 30\] \quad \[-5 \times (-6) = 30\]

**State whether each statement is true or false.**

1. The product of two positive integers is positive.

2. The product of two negative integers is negative.

3. The product of a negative and a positive integer is positive.

4. The product of one negative and two positive integers is negative.

**State whether each product is positive or negative.**

5. \(6 \times 7\)

6. \(-3 \times 4\)

7. \(-5 \times (-2)\)

8. \(8 \times (-8)\)

9. \(-7 \times (-9)\)

10. \(11 \times 4\)

11. \(-3 \times (-12)\)

12. \(2 \times 7\)

13. \(3 \times (-8)\)

**Solve each equation**

14. \(x = -4 \times (-15)\)

15. \(y = -8 \times 7\)

16. \(x = 3 \times (-6)\)

17. \(-4 \times 5 \times 2 = c\)

18. \(3 \times (-9) \times (-2) = d\)

19. \(2 \times (-5) \times (-5) = n\)

20. \(-22 \times (-12) = t\)

21. \(s = -18 \times 32\)

22. \(w = 15 \times (-25)\)
When dividing two integers with different signs, the result is negative.

When dividing two integers with the same sign, the result is positive.

Examples: \[35 \div (-7) = -5\] \[\frac{-30}{5} = -6\]

Examples: \[-35 \div (-7) = 5\] \[\frac{30}{5} = 6\]

State whether each quotient is positive or negative. Then find the quotient

1. \[132 \div 11\]  
2. \[-108 \div 12\]  
3. \[-80 \div (-16)\]

4. \[98 \div (-14)\]  
5. \[30 \div (-3)\]  
6. \[-88 \div 8\]

7. \[-120 \div (-15)\]  
8. \[196 \div 14\]  
9. \[81 \div (-3)\]

State whether each statement is true or false.

10. The quotient of two negative numbers is positive.

11. The quotient of one positive and one negative number is negative.

Divide.

12. \[12 \div (-6)\]  
13. \[-15 \div 3\]  
14. \[14 \div 2\]

15. \[(-21) \div (-7)\]  
16. \[30 \div (-5)\]  
17. \[0 \div 6\]

18. \[64 \div 8\]  
19. \[-81 \div 9\]  
20. \[-49 \div (-7)\]
Dan, Ellie, and Ian each went on a vacation. One went to Mexico, one went to Florida, and one went to Canada. Ian did not go to Florida or Canada. Ellie did not go to Florida. Find where each went for vacation.

Explore  We need to find out where each person went on vacation.

Plan  Make a chart to organize the information. Mark an X to eliminate a possibility.

Solve  Since Ian did not go to Florida or Canada, he must have gone to Mexico. By using the second clue and eliminating possibilities, we find that Ellie went to Canada and that Dan went to Florida.

<table>
<thead>
<tr>
<th></th>
<th>Mexico</th>
<th>Canada</th>
<th>Florida</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dan</td>
<td>X</td>
<td>X</td>
<td>O</td>
</tr>
<tr>
<td>Ellie</td>
<td>X</td>
<td>O</td>
<td>X</td>
</tr>
<tr>
<td>Ian</td>
<td>O</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Examine  Check the results with the clues. There is no conflict with any of the given clues.

Pick an answer by eliminating as many possibilities as possible.

1. 462 765 612 87 223 492
   534 591 688 638 333 692
   a. It is an even number divisible by 6.
   b. It is larger than 400 and smaller than 600.
   c. The sum of the digits is 15.

2. 12 248 76 3 240
   504 7 26 48 25
   86 592 50 9 72
   a. It is not divisible by 6.
   b. It is even.
   c. It is divisible by 4.
   d. The sum of its digits is 14.

3. Theo, Sue, and Lenita each had breakfast. One had toast, one had pancakes, and one had eggs. Use the clues to find out who had what for breakfast.
   a. Lenita did not have pancakes.
   b. Theo did not have eggs or pancakes.

4. Ross is twice as tall as Enrico. Enrico is 6 inches taller than Tony. Which answer could be the heights of the three boys?
   a. Ross, 64 in.; Enrico, 29 in.; Tony, 32 in.
   b. Ross, 32 in.; Enrico, 64 in.; Tony, 67 in.
   c. Ross, 64 in.; Enrico, 32 in.; Tony, 26 in.
**Method:** 1. Identify the variable.
2. To get the variable by itself, add the same number to or subtract the same number from each side of the equation.
3. Check the solution.

**Example:** Solve $x + (-2) = 6$.

\[
x + (-2) = 6
\]
\[
x + (-2) - (-2) = 6 - (-2)
\]
\[
x = 8
\]

**Check:**
\[
x + (-2) = 6
\]
\[
8 + (-2) = 6
\]
\[
6 = 6 \checkmark
\]

**Example:** Solve $x - 9 = -13$.

\[
x - 9 = -13
\]
\[
x - 9 + 9 = -13 + 9
\]
\[
x = -4
\]

**Check:**
\[
x - 9 = -13
\]
\[
-4 - 9 = -13
\]
\[
-13 = -13 \checkmark
\]

**Solve each equation and check your solution. Then graph the solution on the number line.**

1. $x + 5 = 2$ 

2. $11 + w = 10$

3. $a - 7 = -5$

4. $b + (-13) = -13$

5. $-3 + h = -7$

6. $y - (-9) = 12$
Method: 1. Identify the variable.
2. Multiply or divide each side of the equation by the same nonzero number to get the variable by itself.
3. Check the solution.

Example: Solve $-7x = 42$.

$-7x = 42$

$\frac{-7x}{-7} = \frac{42}{-7}$  Divide each side by $-7$.

$x = -6$  The solution is $-6$.

Check: $-7x = 42$

$-7(-6) = 42$

$42 = 42 \checkmark$

Example: Solve $\frac{y}{2} = -2$.

$\frac{y}{2} = -2$

$\frac{y}{2} \cdot (2) = -2 \cdot (2)$  Multiply each side by $2$.

$y = -4$  The solution is $-4$.

Check: $\frac{y}{2} = -2$

$\frac{-4}{2} = -2$

$-2 = -2 \checkmark$

Solve each equation and check your solution. Then graph the solution on the number line.

1. $-3a = 15$

2. $-t = 5$

3. $-1 = \frac{n}{4}$

4. $7r = 28$

5. $0 = \frac{h}{7}$

6. $24 = -8m$

7. $-11b = 44$

8. $\frac{a}{-2} = -1$
A formula shows the relationship between certain quantities. The formula for the distance traveled by a moving object is $d = rt$. In the formula, $d$ represents distance in kilometers (km), $r$ represents the rate in kilometers per hour (km/h), and $t$ represents the time in hours (h).

**Example:** Suppose $r$ is 40 kilometers per hour and $t$ is 3 hours. Find the distance traveled ($d$).

$$d = rt$$

$$d = 40 \times 3 \quad \text{Replace } r \text{ with 40 and } t \text{ with 3.}$$

$$d = 120 \quad \text{The distance traveled is 120 kilometers.}$$

*Use the formula $d = rt$ to find the indicated variables.*

1. $r = 60 \text{ km/h}; t = 4 \text{ h}; d$
2. $d = 100 \text{ km}; t = 2 \text{ h}; r$
3. $r = 55 \text{ km/h}; d = 110 \text{ km}; t$
4. $r = 35 \text{ km/h}; t = 3 \text{ h}; d$
5. $d = 210 \text{ km}; t = 7 \text{ h}; r$
6. $r = 80 \text{ km/h}; d = 320 \text{ km}; t$

The formula $I = \frac{V}{R}$ shows the relationship between the current in amperes ($I$), the voltage in volts ($V$), and the resistance in ohms ($R$) in an electrical circuit.

*Use the formula $I = \frac{V}{R}$ to find the current for each of the following. (Current is measured in amperes.)*

7. $V: 60 \text{ volts}; R: 3 \text{ ohms}$
8. $V: 90 \text{ volts}; R: 3 \text{ ohms}$
9. $V: 100 \text{ volts}; R: 2 \text{ ohms}$
10. $V: 120 \text{ volts}; R: 3 \text{ ohms}$
The perimeter of a rectangle equals twice the sum of the measure of the two sides.

\[ P = a + a + b + b \]
\[ = 2a + 2b, \text{ or } 2(a + b) \]

**Example:**
Find the perimeter of the rectangle.

\[
\begin{array}{c}
\text{7 cm} \\
2 \text{ cm}
\end{array}
\]

\[ P = 2(7 \text{ cm} + 2 \text{ cm}) \]
\[ = 18 \text{ cm} \]

The area of a rectangle equals the product of the measure of the two sides.

\[ A = a \times b \]

**Example:**
Find the area of the rectangle.

\[
\begin{array}{c}
\text{7 cm} \\
2 \text{ cm}
\end{array}
\]

\[ A = 2 \text{ cm} \times 7 \text{ cm} \]
\[ = 14 \text{ cm}^2 \]

**Find the perimeter and area of each rectangle.**

1. \[
\begin{array}{c}
12 \text{ ft} \\
3 \text{ ft}
\end{array}
\]

2. \[
\begin{array}{c}
5 \text{ cm} \\
3 \text{ cm}
\end{array}
\]

3. \[
\begin{array}{c}
15 \text{ yd} \\
10 \text{ yd}
\end{array}
\]

4. \[
\begin{array}{c}
9 \text{ m} \\
6 \text{ m}
\end{array}
\]

5. \[
\begin{array}{c}
7 \text{ in.} \\
3 \text{ in.}
\end{array}
\]

6. \[
\begin{array}{c}
20 \text{ cm} \\
18 \text{ cm}
\end{array}
\]

**Tell whether each expression would give you the perimeter or the area of the rectangle.**

7. \[ xy \]

8. \[(x + 1)(x + 2)\]

9. \[ y + 2x + y \]

10. \[ 4x \]

11. \[ 2(a + b) \]

12. \[ 2(w + 2w) \]
An inequality is a mathematical sentence that contains one of these symbols: <, >, ≤, ≥, or ≠. The meaning of each is given at the right.

The same steps used to solve equations are used to solve inequalities.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;</td>
<td>less than</td>
</tr>
<tr>
<td>&gt;</td>
<td>greater than</td>
</tr>
<tr>
<td>≤</td>
<td>less than or equal to</td>
</tr>
<tr>
<td>≥</td>
<td>greater than or equal to</td>
</tr>
<tr>
<td>≠</td>
<td>not equal to</td>
</tr>
</tbody>
</table>

**Example:** Solve \( x + 6 > 11 \).

\[
x + 6 > 11
\]
\[
x + 6 - 6 > 11 - 6
\]
\[
x > 5
\]

Subtract 6 from each side.

**Check:** Pick a number greater than 5.

Use 8.

\[
x + 6 > 11
\]
\[
8 + 6 > 11
\]
\[
14 > 11
\]

The statement is true, so it checks.

The solution to an inequality with one variable can be shown on a number line. A closed dot is used when the point is included in the solution. An open circle is used when the point is not included in the solution.

**Examples:**

\[
x - 3 \leq -6
\]
\[
x \leq -3
\]

\[
x - 3 > -6
\]
\[
x > -3
\]

The solution to an inequality with one variable can be shown on a number line. A closed dot is used when the point is included in the solution. An open circle is used when the point is not included in the solution.

**Solve each inequality and check your solution. Then graph the solution on the number line.**

1. \(-1 > x + 3\)

\[
-5 -4 -3 -2 -1 0 1 2 3 4 5
\]

2. \(t + 9 \geq 6\)

\[
-5 -4 -3 -2 -1 0 1 2 3 4 5
\]

3. \(-5 \leq r - 3\)

\[
-5 -4 -3 -2 -1 0 1 2 3 4 5
\]

4. \(-7 + m \geq -9\)

\[
-5 -4 -3 -2 -1 0 1 2 3 4 5
\]

5. \(b - 14 < -10\)

\[
-5 -4 -3 -2 -1 0 1 2 3 4 5
\]

6. \(8 + y \geq 6\)

\[
-5 -4 -3 -2 -1 0 1 2 3 4 5
\]
When you multiply or divide each side of an inequality by a positive number, you get a new inequality with the same solutions.

\[
\begin{align*}
3h &< -12 \\
3h ÷ 3 &< -12 ÷ 3 \\
h &< -4
\end{align*}
\]

\[
\begin{align*}
\frac{h}{5} &> 10 \\
\frac{h}{5} ÷ 5 &> 10 ÷ 5 \\
h &> 50
\end{align*}
\]

When you multiply or divide each side by a negative number, you must reverse the inequality symbol. Otherwise, the new inequality will not have the same solutions.

\[
\begin{align*}
-3h &< -12 \\
-3h ÷ (-3) &> -12 ÷ (-3) \\
h &> 4
\end{align*}
\]

\[
\begin{align*}
\frac{h}{-5} &> 10 \\
\frac{h}{-5} ÷ (-5) &< 10 ÷ (-5) \\
h &< -50
\end{align*}
\]

**Do the two inequalities have the same solutions? Write yes or no.**

1. \(2x < 14\)
   - \(x > 7\)  
   - yes

2. \(-x < 0\)
   - \(x > 0\)  
   - yes

3. \(3x < 9\)
   - \(x < 3\)  
   - yes

4. \(-5x > 0\)
   - \(x > 0\)  
   - yes

5. \(-4x < 4\)
   - \(x > -1\)  
   - yes

6. \(-3x > -3\)
   - \(x > 1\)  
   - yes

**Solve each inequality and check your solution.**

7. \(7x < 84\)

8. \(9x > 81\)

9. \(\frac{h}{3} < -10\)

10. \(6p < 12\)

11. \(\frac{h}{4} > -7\)

12. \(0 > -5c\)

13. \(-2d > 4\)

14. \(-2d > -4\)

15. \(-2d < -4\)

16. \(\frac{a}{-3} < 9\)

17. \(\frac{a}{-3} > -9\)

18. \(\frac{a}{3} < -9\)
3-8 Study Guide
Applying Equations and Inequalities

Compare these two examples. On the left, an equation is used to find the solution. On the right, an inequality is used.

**Example:** The record low temperature for Bakersville is -35°F. This is 5 times the record low for Springtown. What is the record low for Springtown?

**Explore** Bakersville's low is 5 times Springtown's.

**Plan** \( B = 5 \times S \)

**Solve** \( -35 = 5 \times S \)

\[
\frac{-35}{5} = S \\
-7 = S
\]

Springtown's low is -7°F.

**Examine** Check the answer in the original problem.

**Example:** The record high temperature for Bakersville is 120°F. This is more than 2 times the record high for Springtown. What is the record high for Springtown?

**Explore** Bakersville's high is more than 2 times Springtown's.

**Plan** \( B > 2 \times S \)

**Solve** \( 120 > 2 \times S \)

\[
\frac{120}{2} > S \\
60 > S
\]

Springtown's high is less than 60°F.

**Examine** Check whether the answer should include less than or more than.

Choose the equation or inequality that could be used to solve each problem.

1. On the way to school it was 10°F. It dropped 13 degrees by the end of the day. What is the temperature at the end of the day?
   A. \( 10 + t = -13 \)  
   B. \( 10 - 13 = t \)  
   C. \( 10 + t < -13 \)  
   D. \( 10 - 13 > t \)

2. Football practice begins in 6 weeks. Ted wants to gain at least 12 pounds. How much must he gain per week?
   A. \( g \leq 6 \times 12 \)  
   B. \( g \leq 12 \div 6 \)  
   C. \( g \geq 6 \times 12 \)  
   D. \( g \geq 12 \div 6 \)

3. The radio Andrea wants costs $16 less than a cassette player. How much does the radio cost?
   A. \( r = c + 16 \)  
   B. \( r = c - 16 \)  
   C. \( r > c + 16 \)  
   D. \( r < c + 16 \)

4. Randy invested $2000 in 150 shares of stock last year. This is twice what the stock is worth today. What is the present value of the stock?
   A. \( p < 2000 \times 2 \)  
   B. \( p > 2000 \div 2 \)  
   C. \( p = 2000 \times 2 \)  
   D. \( p = 2000 \div 2 \)
The factors of a whole number divide that number with no remainder.

3 is a factor of 12 because \(12 \div 3 = 4\) with no remainder.
3 is not a factor of 16 because \(16 \div 3 = 5\) with a remainder of 1.

Because \(12 \div 3 = 4\), with no remainder, we say that 12 is divisible by 3.

A monomial is an integer, a variable, or a product of integers or variables.

Expressions like \(-537\) and \(8ac\) are monomials.
Expressions like \(4t + 5\) and \(-2(3x - 3)\) are not monomials.

**Using divisibility rules, state whether each number is divisible by 2, 3, 5, 6, or 10.**

1. 1060
2. 996
3. 285
4. 705
5. 32
6. 64,230
7. 1645
8. 3241
9. 42,246

**Determine whether each expression is a monomial. Explain why or why not.**

10. \(-3mn\)
11. \(-4x + y\)
12. 672
13. \(65cde\)
14. \(k\)
15. \(-9(3x - 2)\)
Expressions such as \(4^2\), \(a^3\), \(2^n\), and \((x + 3)^5\) are written using exponents. In \(4^2\), the base is 4, and the exponent is 2.

The exponent tells you how many times to use the base as a factor.

\[3^4 = 3 \cdot 3 \cdot 3 \cdot 3, \text{ or } 81\]  
The number named by \(3^4\) is 81.

**Write each product using exponents.**

1. \(5 \cdot 5 \cdot 5\)
2. \(6 \cdot 6 \cdot 6 \cdot 6 \cdot 6\)
3. \(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7\)
4. \(2 \cdot 2 \cdot 2 \cdot 3 \cdot 3\)
5. \(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2\)
6. \(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5\)
7. \(m \cdot m \cdot m \cdot m \cdot m \cdot m\)
8. \(n \cdot n \cdot n \cdot n \cdot n \cdot y \cdot y \cdot y \cdot y\)
9. \(q \cdot q \cdot q \cdot p \cdot p \cdot p \cdot p\)
10. \(2 \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t\)
11. \(9 \cdot 9 \cdot 9 \cdot 9 \cdot 9\)
12. \(4 \cdot 4 \cdot 6 \cdot 6 \cdot x \cdot x \cdot v \cdot v\)

**Write each power as the product of the same factor.**

13. \(5^3\)
14. \(a^5\)
15. \(1^4\)
16. \((-j)^2\)
17. \((y - 2)^2\)
18. \(7^2\)
19. \((-4)^3\)
20. \(5^{22}\)
There are four baseball teams in a single elimination tournament. How many baseball games will be played during the tournament?

**Explore**
There are 4 teams in the single elimination tournament. The problem asks how many games will be played.

**Plan**
Draw a diagram to show how the winning teams will advance through the tournament. A diagram will help to count the number of games.

**Solve**

```
Team A
    \      \      \      \      \\
    Team B  Team C  Team D
```

Notice that 2 games are played in the first round and 1 final game is played for the championship.

\[ 2 + 1 = 3 \]

There will be 3 games played in the tournament.

**Examine***
Each team loses exactly once except the champion. Therefore, \[ 4 - 1 \] or 3 games are played. The answer is correct.

---

**Solve. Use any strategy.**

1. If a coin is tossed three times, how many different combinations of heads and tails are possible?
2. An airport services seven different airlines and nine different types of airplanes. In how many different ways can a person fly out of the airport?

3. Twelve bowlers will be participating in the annual single elimination bowling tournament. How many games will be played during the tournament?
4. A certain buffet restaurant serves six different entrees, eight kinds of beverages, and four types of desserts. How many different combinations for dinner are possible if each includes an entree, a beverage, and a dessert?
A factor tree can be used to find the prime factorization of a composite number. Test prime numbers as factors in order from least to greatest. Test 2, 3, 5, 7, and so on.

\[
\begin{align*}
75 &\quad 30y^2x = 2 \cdot 15 \cdot y^2 \cdot x \\
3 \times 25 &\quad = 2 \cdot 3 \cdot 5 \cdot y \cdot y \cdot x \\
3 \times 5 \times 5 &\quad \text{Are all the factors prime?}
\end{align*}
\]

**Complete each factor tree.**

1. \[
\begin{array}{c}
2 \\
2 \\
2 \\
\end{array}
\]

2. \[
\begin{array}{c}
2 \\
2 \\
2 \\
\end{array}
\]

3. \[
\begin{array}{c}
2 \\
2 \\
2 \\
\end{array}
\]

4. \[
\begin{array}{c}
2 \\
2 \\
2 \\
\end{array}
\]

5. \[
\begin{array}{c}
2 \\
2 \\
2 \\
\end{array}
\]

6. \[
\begin{array}{c}
2 \\
2 \\
2 \\
\end{array}
\]

**Factor completely.**

7. 36

8. -28

9. 50

10. 54xyz

11. 81m^2n

12. 100a^2b^2

13. -164st^3

14. 102f^3g^2

15. -72rt

16. 200xy^3

17. -225w^2vu

18. 140cd^2
Step 1
Factor each number completely.

Step 2
Circle all pairs of factors that the numbers have in common.

Step 3
Find the product of the common factors circled in Step 2.

Find the GCF of 24 and 56.

\[
24 = 2 \cdot 12 = 2 \cdot 2 \cdot 6 = 2 \cdot 2 \cdot 3 \quad 56 = 2 \cdot 28 = 2 \cdot 2 \cdot 14 = 2 \cdot 2 \cdot 2 \cdot 7
\]

\[
24 = 2 \cdot 2 \cdot 3 \quad 56 = 2 \cdot 2 \cdot 2 \cdot 7 \quad 2 \cdot 2 \cdot 2 = 8
\]

8 is the GCF.

Find the GCF of 15\(xy^2\) and 18\(x^2y\).

\[
15xy^2 = 3 \cdot 5 \cdot x \cdot y \cdot y \\
18x^2y = 2 \cdot 9 \cdot x \cdot x \cdot y \\
= 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y
\]

\[
15xy^2 = 3 \cdot 5 \cdot x \cdot y \cdot y \\
18x^2y = 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y
\]

3xy is the GCF.

Find the GCF of each set of numbers or monomials.

1. 8, 10
2. 15, 24
3. 42, 54

4. 22, 55
5. 21, 49
6. 75, 100

7. 8n, 18n
8. 15vw^2, 27v^3w
9. 125ab, 200a^2b^2

10. 26r^2, 91s^2, 13rs
11. 48xy, 72x^3y^3
12. 15m^3n^2, 30mn, 135m^2n^2
Write each fraction in simplest form. If the fraction is already in simplest form, write simplified.

1. \( \frac{14}{20} \)
2. \( \frac{16}{35} \)
3. \( \frac{16}{20} \)
4. \( \frac{10}{40} \)
5. \( \frac{16}{36} \)
6. \( \frac{45}{48} \)
7. \( \frac{23}{55} \)
8. \( \frac{49}{56} \)
9. \( \frac{13}{26} \)
10. \( \frac{3x^3}{12x^4} \)
11. \( \frac{9x^2}{16y^3} \)
12. \( \frac{4p^2q^3}{12pq} \)
13. \( \frac{3ab}{15a^2b^3} \)
14. \( \frac{-3p^2s^5}{-12r^3s^6} \)
15. \( \frac{7m^3n^{12}}{15mn^8} \)
16. \( \frac{-13p^4q^3}{26p^4q^3} \)
17. \( \frac{12a^4m^3}{16a^3m^8} \)
18. \( \frac{6q^3v^3}{18q^2v^4} \)
Follow these steps to find the least common multiple (LCM) of two or more numbers or algebraic expressions.

**Step 1**
Factor each number or monomial completely.

**Step 2**
Write the prime factorization as powers.

**Step 3**
Multiply the greatest power of each number or variable to find the LCM.

Find the LCM of $30a$ and $45b^2$.

- $30a = 2 \cdot 3 \cdot 5 \cdot a$
- $45b^2 = 3 \cdot 3 \cdot 5 \cdot b \cdot b$
- $LCM = 2^1 \cdot 3^2 \cdot 5^1 \cdot a^1 \cdot b^2$
- $= 2 \cdot 9 \cdot 5 \cdot a \cdot b^2$
- $= 90ab^2$

You can use the LCM to compare two fractions. To compare two fractions, write them as equal fractions with the same denominator by multiplying or dividing the numerator and denominator by the same nonzero number.

Which is greater, $\frac{3}{5}$ or $\frac{2}{3}$?

\[
\begin{array}{c c}
\times 3 & \times 5 \\
3 & = 9 \\
5 & = 15 \\
\end{array}
\]

\[
\begin{array}{c c}
\times 3 & \times 5 \\
2 & = 10 \\
3 & = 15 \\
\end{array}
\]

Since $\frac{9}{15} < \frac{10}{15}$, $\frac{3}{5} < \frac{2}{3}$.

**Find the LCM of each set of numbers or algebraic expressions.**

1. 4, 6
2. 6, 9
3. 21, 9
4. 6, 10
5. 20, 25
6. 15, 20
7. 12, 36
8. 20, 30
9. $4x$, $3x$
10. $3c^2$, $7c$
11. $16w^2$, 72
12. $4f$, $10f^2$, $12f^2$

**Write < or > in each box to make a true statement.**

13. $\frac{2}{3} \not< \frac{6}{24}$
14. $\frac{2}{5} \not> \frac{3}{4}$
15. $\frac{5}{8} \not< \frac{5}{6}$
16. $\frac{7}{10} \not< \frac{3}{4}$
17. $\frac{4}{6} \not< \frac{3}{12}$
18. $\frac{6}{9} \not< \frac{2}{7}$
To multiply powers with the same base, add the exponents.

**Examples:**

\[ x^4 \cdot x^8 = x^{4+8} = x^{12} \]
\[ (4x^2)(2x^3) = (4 \cdot 2)(x^2 \cdot x^3) = 8(x^{2+3}) = 8x^7 \]

To divide powers with the same base, subtract the exponents.

**Examples:**

\[ \frac{4^8}{4^5} = 4^{8-5} = 4^3 \]
\[ \frac{x^5}{x^3} = x^{5-3} = x^2 \]

---

**Find each product or quotient. Express your answer in exponential form.**

1. \( 8^2 \cdot 8^3 \)
2. \( x^5 \cdot x^1 \)
3. \( 10^2 \cdot 10^7 \)

4. \( (3x^2)(2x^4) \)
5. \( y^3(y^2x) \)
6. \( a^3 \cdot a^3 \)

7. \( n^1 \cdot n^3 \)
8. \( 20^3 \cdot 20^5 \)
9. \( (4x^3)(-2x^5) \)

10. \( \frac{a^5}{a^3} \)
11. \( \frac{10^4}{10^2} \)
12. \( \frac{h^8}{h^4} \)

13. \( \frac{t^9}{t^3} \)
14. \( \frac{(-d)^3}{(-d)^2} \)
15. \( \frac{c^3}{c^1} \)

16. \( \frac{x^4x^3}{x^5} \)
17. \( \frac{5^6}{5^3} \)
18. \( \frac{6^7}{6^2} \)
Using the definition, \(10^{-6} = \frac{1}{10}\) and \(x^{-2} = \frac{1}{x}\).

The definition also shows that \(\frac{1}{10} = 10^{-6}\) and \(\frac{1}{x} = x^{-2}\).

**Represent each expression using positive exponents.**

1. \(3^{-1}\)
2. \(e^{-4f}\)
3. \(w^{-2}\)
4. \(\frac{1}{5^{-2}}\)
5. \((-4)^{-3}\)
6. \(7(xy)^{-1}\)

**Write each fraction as an exponent with negative exponents.**

7. \(\frac{b}{a^3}\)
8. \(\frac{6}{2^3}\)
9. \(\frac{1}{6^3}\)
10. \(\frac{1}{100}\)
11. \(\frac{1}{u}\)
12. \(\frac{s}{r^3t^2}\)

**Evaluate each expression.**

13. \(2^x\) if \(x = -4\)
14. \((3b)^{-3}\) if \(b = -2\)
15. \(5w^{-2}\) if \(w = 3\)
The set of **whole numbers** includes 0, 1, 2, 3, . . . .

The set of **integers** includes . . . , −2, −1, 0, 1, 2, . . . .

Any number that can be written as a fraction is called a **rational number**.

To express a decimal as a fraction or mixed number, write the digits of the decimal as the numerator and use the appropriate power of ten (10, 100, 1000, . . .) as the denominator. Simplify if necessary.

**Examples:** Express each decimal as a fraction in simplest form.

a. $0.125 = \frac{125}{1000} = \frac{1}{8}$

So, $0.125 = \frac{1}{8}$.

b. $0.7$

Let $N = 0.777 \ldots$

Then $10N = 7.777 \ldots$

$10N = 7.777 \ldots$

$-1N = 0.777 \ldots$

$9N = 7$

$N = \frac{7}{9}$

**Express each decimal as a fraction or mixed number in simplest form.**

1. 0.8
2. −0.4
3. 0.09

4. 0.48
5. 0.15
6. 0.25

7. −0.81
8. 0.88
9. 0.6

10. 0.845
11. 5.36
12. 7.16

**Name the set(s) of numbers to which each number belongs.**

(Use the symbols $W =$ whole numbers, $I =$ integers, and $R =$ rationals.)

13. 2
14. −4
15. 4.169
16. $\frac{1}{4}$

17. $-2\frac{3}{5}$
18. 0.32
19. $\frac{18}{3}$
20. −8.0
You can use rounding to estimate sums and differences. Round each number to a convenient place-value position.

\[ 4.25 + 3.56 \quad \quad \$187.45 - 53.81 \]
\[ 4.25 \rightarrow 4 + 4 = 8 \quad \quad \$187.45 - 53.81 \rightarrow 190 - 50 = 140 \]
\[ 4.25 + 3.56 \text{ is about } 8. \quad \quad \$187.45 - 53.81 \text{ is about } $140. \]

To estimate sums and differences of mixed numbers, round each mixed number to the nearest whole number. To estimate sums and differences of proper fractions, round each fraction to 0, \( \frac{1}{2} \), or 1.

\[ \frac{15}{19} - \frac{7}{43} \rightarrow 16 - 7 = 9 \quad \quad \frac{12}{25} + \frac{7}{65} + \frac{34}{35} \rightarrow \frac{1}{2} + 0 + 1 = 1 \frac{1}{2} \]

**Round to the nearest whole number.**

1. 9.23  
2. 3.045  
3. 17.792  
4. 634.572  

5. 37\( \frac{5}{29} \)  
6. 6\( \frac{5}{6} \)  
7. 13\( \frac{1}{7} \)  
8. 4\( \frac{8}{9} \)

**Round each fraction to 0, \( \frac{1}{2} \), or 1.**

9. \( \frac{17}{35} \)  
10. \( \frac{5}{9} \)  
11. \( \frac{2}{11} \)  
12. \( \frac{15}{16} \)

**Estimate each sum or difference.**

13. 24.02 + 17.46  
14. 74.63 - 65.89  
15. \( 10\frac{17}{20} - 8\frac{2}{9} \)

16. \( 28\frac{1}{6} - 4\frac{9}{17} \)  
17. \( 2\frac{4}{11} + \frac{6}{31} \)  
18. \( 21\frac{6}{7} + 5\frac{8}{27} \)
To add or subtract decimals, first align the decimal points and place zeros where necessary. Then add or subtract as with whole numbers.

Solve \( m = 38.5 + 52.12 \rightarrow 38.50 \) \( + 52.12 \)
\[ m = 90.62 \]

Solve \( d = 235 - 13.8 \rightarrow 235.0 \) \( - 13.8 \)
\[ d = 221.1 \]

**Solve each equation.**

1. \( x = 3.2 + 4.7 \)
2. \( -2.06 + 3.15 = m \)
3. \( y = -36.09 - (-7.01) \)
4. \( 47.9 + 3.24 = w \)
5. \( g = 0.5623 - 0.3541 \)
6. \( 52.5 + 8.62 = k \)
7. \( h = -27.8 + (-14.32) \)
8. \( 3.09 - 0.05 = b \)
9. \( m = 16.2 - 5.59 \)
10. \( 58 - 0.232 = z \)
11. \( j = 23 + (-1.59) \)
12. \( 15.6 - 0.423 = g \)
13. \( r = -8.52 + 2.43 \)
14. \( 150 - 25.6 = f \)
15. \( v = -3.56 - 0.49 \)

**Simplify each expression.**

16. \( 0.5y + 0.7y \)
17. \( 9x - 2.5x \)
18. \( 3.81n + 0.092n + 4.6n \)
19. \( 3.56t - 0.59t + 46.08t \)
20. \( 3.0w - 24.15w + 56.052w \)
21. \( 215.5v - 36v - 4.63v \)
5-4 Study Guide
Adding and Subtracting Like Fractions

To add fractions with like denominators, add the numerators. Write the sum over the common denominator. When the sum of the two fractions is greater than 1, the sum is written as a mixed number.

\[ \begin{align*}
w &= \frac{4}{12} + \frac{10}{12} \\
&= \frac{14}{12} \\
&= 1\frac{2}{12} \text{ or } 1\frac{1}{6}
\end{align*} \]

To subtract fractions with like denominators, subtract the numerators. Write the difference over the common denominator.

\[ \begin{align*}
x &= \frac{7}{15} - \frac{1}{15} \\
x &= \frac{6}{15} \text{ or } \frac{2}{5}
\end{align*} \]

\[ \begin{align*}
y &= \frac{4}{4} - \frac{2}{4} \\
y &= \frac{2}{4} - \frac{1}{2}
\end{align*} \]

Solve each equation. Write the solution in simplest form.

1. \( \frac{4}{7} + \frac{2}{7} = a \)
2. \( m = \frac{1}{9} + \frac{2}{9} \)
3. \( s = \frac{12}{20} + \frac{7}{20} \)

4. \( \frac{15}{16} + \frac{7}{16} = n \)
5. \( \frac{13}{20} - \frac{3}{20} = v \)
6. \( d = \frac{23}{18} - \frac{15}{18} \)

7. \( -\frac{12}{50} + \left( -\frac{2}{50} \right) = h \)
8. \( j = \frac{13}{16} - \frac{7}{16} \)
9. \( \frac{62}{52} - \frac{12}{52} = f \)

10. \( \frac{11}{8} - \frac{17}{8} = g \)
11. \( \frac{17}{32} - \frac{5}{32} = d \)
12. \( b = \frac{15}{42} + \left( \frac{11}{42} \right) \)

13. \( \frac{2}{30} - \frac{12}{30} = y \)
14. \( c = \frac{6}{7} + \frac{6}{7} \)
15. \( p = \frac{7}{8} - \frac{3}{8} \)

Evaluate each expression if \( x = \frac{1}{15} \), \( y = \frac{8}{15} \), and \( z = \frac{4}{15} \). Write in simplest form.

16. \( x - y \)
17. \( z - x \)
18. \( x + z \)
19. \( x + y \)
20. \( y - z \)
21. \( y + z \)
To find the sum or difference of two fractions with unlike denominators, rename each fraction with a common denominator. The common denominator will be the least common multiple of the given denominators. This is called the least common denominator (LCD).

List the multiples to find the LCD. Rename each fraction with the LCD. Add or subtract. Simplify if necessary.

\[
\begin{align*}
\text{List the multiples:} & \quad 9: 9, 18, 27, \ldots & \quad 6: 6, 12, 18, \ldots \\
\text{Rename each fraction:} & \quad \frac{1}{9} = \frac{2}{18} & \quad \frac{2}{6} = \frac{6}{18} \\
\text{Add or subtract:} & \quad \frac{1}{9} + \frac{2}{6} = \frac{2}{18} & \quad \frac{8}{18} \text{ or } \frac{4}{9}
\end{align*}
\]

LCM: 18

Solve each equation. Write the solution in simplest form.

1. \( \frac{1}{3} - \frac{1}{6} = c \)
2. \( \frac{1}{5} + \frac{1}{7} = k \)
3. \( b = \frac{1}{8} + \frac{1}{9} \)
4. \( \frac{7}{16} - \frac{3}{8} = a \)
5. \( g = \frac{7}{10} + \frac{2}{5} \)
6. \( \frac{3}{14} - \frac{1}{7} = h \)
7. \( \frac{5}{12} + \frac{1}{3} = d \)
8. \( \frac{5}{4} + \frac{3}{2} = b \)
9. \( \frac{1}{6} - \frac{3}{8} = s \)
10. \( a = \frac{9}{6} + \frac{7}{9} \)
11. \( 11\frac{3}{16} - 5\frac{1}{12} = m \)
12. \( 18\frac{7}{30} - 3\frac{1}{6} = y \)

Evaluate each expression if \( c = \frac{2}{3}, \ d = \frac{3}{4}, \ \text{and} \ f = 2\frac{5}{6}. \) Write the solution in simplest form.

13. \( f + c \)
14. \( d + f \)
15. \( c - d \)
16. \( f - c \)
17. \( c + d \)
18. \( d + f + c \)
19. \( f - d + c \)
20. \( f + d - c \)
21. \( c + f - d \)
An equation like \( x - 6.2 = 13.4 \) can be solved using addition.

\[
x - 6.2 = 13.4
\]

\[
x - 6.2 + 6.2 = 13.4 + 6.2 \quad \text{Add 6.2 to each side.}
\]

\[
x = 19.6
\]

To check the solution, replace \( x \) with 19.6

\[
x - 6.2 = 13.4
\]

\[
19.6 - 6.2 \neq 13.4
\]

\[
13.4 = 13.4 \checkmark \quad \text{The solution is 19.6.}
\]

An equation like \( s + \frac{2}{3} = \frac{5}{2} \) can be solved using subtraction.

\[
s + \frac{2}{3} = \frac{5}{2}
\]

\[
s + \frac{2}{3} - \frac{2}{3} = \frac{5}{2} - \frac{2}{3}
\]

\[
s = \frac{11}{6}
\]

To check the solution, replace \( s \) with \( \frac{11}{6} \).

\[
s + \frac{2}{3} = \frac{5}{2} \rightarrow \frac{11}{6} + \frac{2}{3} = \frac{5}{2}
\]

\[
\frac{15}{6} = \frac{5}{2}
\]

\[
\frac{5}{2} = \frac{5}{2} \checkmark \quad \text{The solution is } \frac{11}{16}.
\]

**Solve each equation. Check your solution.**

1. \( x + 5.3 = 19.6 \)
2. \( q - 1.8 = 4.25 \)
3. \( x + 9.16 = -10.16 \)

4. \( -35.2 = n - 16 \)
5. \( m - 4.3 = -7.5 \)
6. \( 0.45 + p = 1.35 \)

7. \( \frac{6}{5} = n + \frac{4}{10} \)
8. \( \frac{1}{3} + w = \frac{2}{5} \)
9. \( \frac{5}{2} = a - \frac{3}{2} \)

10. \( h - 10.6 = 7.3 \)
11. \( d + 5.8 = 4.17 \)
12. \( a - \frac{2}{4} = 6\frac{1}{8} \)

13. \( q - 0.24 = 32.15 \)
14. \( c + \frac{5}{12} = \frac{5}{6} \)
15. \( 62.03 + j = 78.2 \)

16. \( b - \frac{7}{2} = \frac{1}{2} \)
17. \( 14.7 = k - 17.3 \)
18. \( 0.75 + c = 2.81 \)
Solving Inequalities

An equation like \( b + \frac{3}{7} \geq 2 \) can be solved using subtraction.

\[
\begin{align*}
\quad b + \frac{3}{7} & \geq 2 \\
\quad b + \frac{3}{7} - \frac{3}{7} & \geq 2 - \frac{3}{7} \quad \text{Subtract} \; \frac{3}{7} \; \text{from each side.} \\
\quad b & \geq \frac{14}{7} - \frac{3}{7} \\
\quad b & \geq \frac{11}{7} \quad \text{or} \; \frac{14}{7}
\end{align*}
\]

Check the solution, replacing \( b \) with a number greater than \( \frac{14}{7} \). Try 2.

\[
\begin{align*}
\quad b + \frac{3}{7} & \geq 2 \\
\quad 2 + \frac{3}{7} & \geq 2 \\
\quad \frac{14}{7} & \geq 2 \quad \checkmark \; \text{The solution is} \; b \geq \frac{14}{7}.
\end{align*}
\]

An equation like \( d - 2.4 \leq 5.3 \) can be solved using addition.

\[
\begin{align*}
\quad d - 2.4 & \leq 5.3 \\
\quad d - 2.4 + 2.4 & \leq 5.3 + 2.4 \\
\quad d & \leq 7.7
\end{align*}
\]

Check the solution, replacing \( d \) with number less than or equal to 7.7. Try 7.4.

\[
\begin{align*}
\quad d - 2.4 & \leq 5.3 \\
\quad 7.4 - 2.4 & \leq 5.3 \\
\quad 5 & \leq 5.3 \quad \checkmark \; \text{The solution is} \; d \leq 7.7.
\end{align*}
\]

Solve each inequality and check your solution. Then graph the solution on the number line.

1. \( w - 1\frac{3}{4} \geq \frac{5}{12} \)

\[
\begin{array}{cccccccc}
\quad -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\quad w & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad
\end{array}
\]

2. \( y - 2.4 \leq -4.9 \)

\[
\begin{array}{cccccccc}
\quad -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\quad y & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad
\end{array}
\]

3. \( \frac{2}{3} + r < \frac{5}{8} \)

\[
\begin{array}{cccccccc}
\quad -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\quad \frac{2}{3} + r & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad
\end{array}
\]

4. \( b + 7\frac{1}{2} < 3\frac{1}{2} \)

\[
\begin{array}{cccccccc}
\quad -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\quad b + 7\frac{1}{2} & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad
\end{array}
\]

5. \( 53.60 + m > 49.10 \)

\[
\begin{array}{cccccccc}
\quad -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\quad 53.60 + m & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad
\end{array}
\]

6. \( x + \frac{1}{6} \leq 3\frac{3}{5} \)

\[
\begin{array}{cccccccc}
\quad -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\quad x + \frac{1}{6} & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad
\end{array}
\]

7. \( -5.8 \leq n - 2.3 \)

\[
\begin{array}{cccccccc}
\quad -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\quad -5.8 & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad
\end{array}
\]

8. \( 0.75 + c > 2.81 \)

\[
\begin{array}{cccccccc}
\quad -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\quad 0.75 + c & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad
\end{array}
\]
To make a conclusion based on what has happened in the past is called **inductive reasoning**.

**Example:** Carla noticed that for the last four Wednesdays her teacher has given a pop quiz. So, Carla assumed that on Wednesday there will be a pop quiz.

**Deductive reasoning** uses a rule to make a conclusion or a decision.

**Example:** The radius of a circle is half its diameter. Circle $G$ has a diameter 12 meters long. Therefore, the measure of the radius must be 6 meters.

**State whether each is an example of inductive or deductive reasoning. Explain your answer.**

1. The school cafeteria has served pizza every Tuesday for five weeks. Jessica says, “Tomorrow is Tuesday. We will probably have pizza.”

2. Teams that win seven games will make the playoffs. Carter Junior High won seven games, so they will go to the playoffs.

3. It has rained on the first day of school for the past three years. Kimo thinks that it will rain on the first day of school this year.

4. Any number multiplied by zero is equal to zero. $408,249 \times 0 = 0$

5. If a student earns an 90% or higher on his or her term paper, then the paper will be entered in the literature contest. Andre earned an 98%, so his paper will be entered in the literature contest.

6. Every customer who came into Balmer’s Clothing was carrying an umbrella. Mrs. Balmer decided that it was probably raining.
5-9 Study Guide
Integration: Discrete Mathematics
Arithmetic Sequences

The list of numbers 4, 11, 18, 25, 32, 39, . . . is called a sequence.

Each term is 7 more than the previous term. That is, there is a common difference of 7.

11 − 4 = 7, 18 − 11 = 7, 25 − 18 = 7, . . .

Since the difference between any two consecutive terms is the same, the sequence is called an arithmetic sequence.

State whether each sequence is an arithmetic sequence. Then write the next three terms of each sequence.

1. 5, 14, 23, 32, 41, . . .
2. 24, 39, 54, 69, 84, . . .
4. 0.50, 0.54, 0.58, 0.62, 0.66, . . .
5. \(4 \frac{1}{3}, \frac{5}{3} \), 7, \(8 \frac{1}{3}, 9 \frac{2}{3}\) . . .
6. \(2 \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\) . . .
7. 112, 125, 139, 154, . . .
8. 0.3, 0.6, 0.9, . . .
9. 4, 4, 4, 4, . . .
10. 2.06, 2.12, 2.18, 2.24, . . .
11. \(53 \frac{3}{10}, 55 \frac{4}{5}, 58 \frac{3}{10}\) . . .
12. \(6 \frac{1}{2}, 7 \frac{1}{12}, 7 \frac{2}{3}\) . . .
To change a fraction to a decimal, divide the numerator by the
denominator. Stop when a remainder of zero is obtained or a
pattern develops in the quotient.

**Examples:** Change each fraction to a decimal.

\[
\begin{align*}
\text{a. } & \quad \frac{13}{20} = 0.65 \\
& \quad \begin{array}{c}
13.00 \\
-120 \\
100 \\
-100 \\
0
\end{array} \\
\text{b. } & \quad \frac{1}{6} = 0.166 \ldots \\
& \quad \begin{array}{c}
1.000 \\
-0.6 \\
-0.4 \\
-0.36 \\
0
\end{array}
\end{align*}
\]

\[
\frac{13}{20} \text{ is equivalent to the terminating decimal } 0.65. \quad \frac{1}{6} \text{ is equivalent to the repeating decimal } 0.16. \\
\]

On a number line, numbers are in order of size (magnitude).

\[
\begin{array}{c}
\frac{4}{5} < \frac{5}{6}\end{array}
\]

\[
\begin{array}{c}
\text{Lesser} \\
-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
\text{Greater}
\end{array}
\]

**Write each fraction as a decimal. Use a bar to show a repeating decimal.**

1. \( \frac{1}{4} \) 
2. \( \frac{17}{20} \) 
3. \( \frac{3}{8} \) 
4. \( \frac{2}{9} \) 
5. \( \frac{19}{25} \) 
6. \( \frac{5}{6} \) 
7. \( \frac{4}{5} \) 
8. \( \frac{5}{8} \) 
9. \( \frac{33}{40} \) 
10. \( \frac{7}{16} \) 
11. \( \frac{11}{15} \) 
12. \( \frac{5}{12} \)

**Write > or < in each blank to make a true sentence.**

13. \( \frac{41}{4} \quad \_ \quad 4.13 \) 
14. \( \frac{2}{3} \quad \_ \quad \frac{3}{5} \) 
15. \( \frac{32}{7} \quad \_ \quad -2 \frac{1}{4} \) 
16. \( \frac{4}{11} \quad \_ \quad \frac{7}{12} \) 
17. \( -7 \frac{1}{8} \quad \_ \quad -7.34 \) 
18. \( 1 \frac{6}{7} \quad \_ \quad 1.893 \)
Estimate the products and quotients of rational numbers by using rounding and compatible numbers.

\[26.5 \div 4.16 \rightarrow 28 \div 4 \text{ or } 7\]

Even though 26.5 rounds to 27, 28 is a compatible number because 28 is divisible by 4.

\[3 \times 47.98 \rightarrow 3 \times 50 \text{ or } 150\]

\[\frac{4}{9} \times 20 \rightarrow \frac{1}{2} \times 20 \text{ or } 10\]

\[\frac{4}{9} \text{ is close to } \frac{1}{2}\]

\[25 \div 4 \frac{5}{6} \rightarrow 25 \div 5 \text{ or } 5\]

\[4 \frac{5}{6} \text{ is close to } 5.\]

Estimate each product or quotient.

1. \(27.2 \times 8.1\)
2. \(9.32 \times 6.5\)
3. \(19.1 \div 3.6\)

4. \((8.53)(4.86)\)
5. \(75.61 \div 1.9\)
6. \(24.6 \div 4.8\)

7. \(\frac{1}{3} \times 23\)
8. \(\left(\frac{1}{9}\right)(35)\)
9. \(\frac{3}{7} \times 12\)

10. \(\frac{7}{16} \times 240\)
11. \(\frac{6}{10} \times 28\)
12. \(16 \times \frac{21}{48}\)

13. \(45 \div 8 \frac{6}{7}\)
14. \(315 \div 4 \frac{11}{12}\)
15. \(\frac{29}{54} \times 304\)

16. \(400 \times \frac{11}{21}\)
17. \(156 \div 12 \frac{14}{15}\)
18. \(46 \div 1 \frac{32}{37}\)

19. \(14.7 \div 5.03\)
20. \(48.9 \div 24.8\)
21. \(68.04 \div 0.96\)
To multiply two fractions, first multiply the numerators. Then, multiply the denominators. Write the product in simplest form.

\[
\frac{1}{2} \times \frac{3}{5} = \frac{1 \times 3}{2 \times 5} = \frac{3}{10}
\]

To multiply with mixed numbers, first change the mixed numbers to improper fractions. Then multiply the fraction.

Solve each equation. Write the solution in simplest form.

1. \( t = \frac{1}{2} \times \left( -\frac{1}{3} \right) \)
2. \( \frac{3}{4} \times \frac{1}{2} = f \)
3. \( c = \frac{2}{3} \times \left( -\frac{1}{5} \right) \)
4. \( \frac{1}{3} \times \frac{2}{7} = d \)
5. \( n = \frac{1}{10} \times \left( -\frac{3}{6} \right) \)
6. \( \frac{4}{9} \times \frac{6}{7} = b \)
7. \( \frac{6}{7} \times \frac{1}{12} = w \)
8. \( \frac{2}{9} \times \frac{4}{5} = q \)
9. \( m = \frac{3}{3} \times \frac{2}{4} \)
10. \( y = \frac{3}{4} \times \left( -\frac{1}{2} \right) \)
11. \( t = \frac{5}{6} \times \left( -\frac{1}{14} \right) \)
12. \( \frac{3}{4} \times \frac{2}{3} = r \)

Evaluate each expression if \( a = \frac{3}{4}, \ x = \frac{1}{2}, \) and \( y = \frac{3}{7} \)

13. \( a^2 \)
14. \( 2ax \)
15. \( ay \)
16. \( xy \)
17. \( \frac{1}{2}a \)
18. \( 3y \)
When dividing with fractions, you will use the reciprocal of a number. Two numbers whose product is 1 are called **reciprocals**. The reciprocal is also called the **multiplicative inverse**.

The reciprocal of 4 is \( \frac{1}{4} \) because \( 4 \times \frac{1}{4} = 1 \).

The reciprocal of \( \frac{4}{5} \) is \( \frac{5}{4} \) because \( \frac{4}{5} \times \frac{5}{4} = 1 \).

To divide by a rational number, multiply by its multiplicative inverse (its reciprocal).

If dividing mixed numbers, first rename the mixed numbers as fractions. Then, divide.

\[
4 \div \frac{1}{2} = 4 \times 2 = 8 \\
-1\frac{1}{5} \div 2\frac{2}{5} = -\frac{6}{5} \div \frac{12}{5} = \frac{-6 \times 5}{12} = \frac{-30}{60} = \frac{1}{2}
\]

Name the multiplicative inverse for each rational number.

1. \( \frac{1}{5} \)  
2. \( \frac{3}{5} \)  
3. \( \frac{-7}{2} \)  
4. \( \frac{5}{6} \)

5. \( 3 \)  
6. \( -7 \)  
7. \( \frac{-1}{8} \)  
8. \( \frac{1}{10} \)

Solve each equation. Write the solution in simplest form.

9. \( \frac{1}{4} \div \left( \frac{-2}{5} \right) = r \)  
10. \( \frac{3}{8} \div \frac{4}{5} = j \)  
11. \( y = \frac{-2}{9} \div \frac{3}{5} \)

12. \( m = 10 \div \frac{2}{3} \)  
13. \( t = \frac{-1}{3} \div (-4) \)  
14. \( n = \frac{2}{3} \div \left( \frac{1}{2} \right) \)

15. \( -1\frac{3}{5} \div \frac{1}{2} = q \)  
16. \( 4\frac{1}{9} \div \frac{5}{6} = x \)  
17. \( \frac{3}{5} \div \frac{1}{2} = d \)

18. \( p = 1\frac{3}{4} \div 2\frac{2}{3} \)  
19. \( f = -2\frac{1}{2} \div \left( \frac{-14}{5} \right) \)  
20. \( 1\frac{1}{4} \div 4\frac{1}{8} = w \)
When multiplying decimals, the number of decimal places in the product is the same as the sum of the decimal places in the factors.

**Example:** \( c = (-8.4)(0.62) \)

\[
\begin{align*}
-8.4 & \quad \text{1 decimal place} \\
\times 0.62 & \quad \text{2 decimal places} \\
\hline
168 & \\
\hline
+504 & \\
\hline
-5.208 & \quad \text{3 decimal places}
\end{align*}
\]

The sum of the decimal places in the factors is 3, so the product has 3 decimal places. \( c = -5.208 \)

To divide by a decimal, first multiply both the divisor and dividend by a power of ten so that the divisor is a whole number. Then divide as with whole numbers.

**Example:** \( 0.8 \div 12.96 \rightarrow \) Since 0.8 has 1 decimal place, multiply 0.8 and 12.96 by 10.

\[
\begin{align*}
0.8 & \quad \text{1 decimal place} \\
\times 12.96 & \rightarrow \text{16.2} \\
\end{align*}
\]

Solve each equation.

1. \((66.3)(0.04) = k\)  
2. \((4.1)(-12.2) = p\)  
3. \(x = (-84)(-2.4)\)

4. \((-34.7)(-3) = m\)  
5. \(c = (80.4)(0.02)\)  
6. \((-7.19)(3.9) = t\)

7. \(d = (1.94)(18)\)  
8. \((23.5)(-0.7) = f\)  
9. \(n = (4283)(-1.4)\)

10. \(0.184 \div 8 = b\)  
11. \(d = 0.18 \div 0.9\)  
12. \(c = 3.066 \div (-0.3)\)

13. \(-0.045 \div 0.5 = f\)  
14. \(w = 40.05 \div (-4.5)\)  
15. \(135 \div (-0.15) = q\)

16. \(-30.91 \div (-11) = x\)  
17. \(27.606 \div 0.086 = f\)  
18. \(t = -0.992 \div 8\)
The heights of the school ensemble members are listed at the left.

The **mean** height is the sum of all the heights divided by the number of addends.

\[
\frac{57 + 60 + 61 + 62 + 62 + 70}{6} = \frac{372}{6} \text{ or } 62
\]

The mean is 62.

The **median** height is the middle number when the data are listed in order. Since there are two middle numbers, 61 and 62, the median is the mean of these two. The median is 61.5.

The **mode** is the height that appears most often. The mode is 62.

---

**List each set of data from least to greatest. Then find the mean, median, and mode. When necessary, round to the nearest tenth.**

1. 98, 63, 51, 52, 99, 57, 54, 99
2. 69, 68, 65, 64, 68, 69, 68, 67
3. 73, 75, 71, 69, 72, 71, 73, 71
4. 14, 11, 12, 13, 14, 15, 16, 13, 12, 13

---

**Solve**

5. Jim’s math quiz scores were 87, 79, 100, 83, and 88. Find the mean of his scores.

6. The high temperatures for a week in May were 68, 70, 68, 66, 70, 74, and 72. Find the median, mean, and mode.
To solve equations containing rational numbers, multiply each side by the same number to get the variable by itself.

\[
-\frac{y}{5} = 8 \\
\frac{-1}{5} \cdot y = 8
\]

The number you multiply each side by is the reciprocal of the number that is multiplied times the variable.

\[
-\frac{5}{1} \cdot \frac{1}{5} \cdot y = \frac{-5}{1} \cdot 8 \\
y = -40
\]

Use the same method to solve inequalities containing rational numbers.

\[
-\frac{2}{3} x \leq \frac{1}{2} \\
-\frac{3}{2} \cdot \frac{2}{3} x \leq \frac{1}{2} \cdot \frac{3}{2} \\
x \geq -\frac{3}{4}
\]

When you multiply or divide each side of an inequality by a negative number, you must reverse the inequality sign.

**Write the number you would multiply each side by in order to get the variable by itself.**

1. \(4n = 12\)  
2. \(\frac{a}{2} = 50\)  
3. \(-7t = 21\)  
4. \(-\frac{m}{6} = 2\)

5. \(-35 < \frac{h}{7}\)  
6. \(0.25b < 5\)  
7. \(2.4 \leq -6x\)  
8. \(-\frac{c}{3} \geq -12\)

**Solve each equation or inequality. Check your solution.**

9. \(4n = 12\)  
10. \(\frac{a}{2} = 50\)  
11. \(-7t = 21\)  
12. \(-\frac{m}{6} = 2\)

13. \(4n \leq 12\)  
14. \(\frac{a}{2} > 50\)  
15. \(-7t > 21\)  
16. \(-\frac{m}{6} \leq 2\)

17. \(-35 = \frac{h}{7}\)  
18. \(0.25b = 5\)  
19. \(2.4 = -6x\)  
20. \(-\frac{c}{3} = -12\)

21. \(-35 \geq \frac{h}{7}\)  
22. \(0.25b < 5\)  
23. \(2.4 < -6x\)  
24. \(-\frac{c}{3} > -12\)
Two lists of numbers, or sequences, are shown below.

3, 6, 12, 24, 48, \cdots \quad \text{Each number in the pattern is } 2 \text{ times the number before it.}

162, 54, 18, 6, 2, \cdots \quad \text{Each number in the pattern is } \frac{1}{3} \text{ times the number before it.}

In each sequence above, each number after the first can be obtained from the previous one by multiplying it by a fixed number called the common ratio. Such a sequence is called a geometric sequence. The common ratios above are 2 and \frac{1}{3}, respectively.

\[
\begin{array}{cccc}
  \frac{6}{3} & = 2 & \frac{12}{6} & = 2 \\
  \frac{54}{162} & = \frac{1}{3} & \frac{18}{54} & = \frac{1}{3} \\
  \frac{24}{12} & = 2 & \frac{6}{18} & = \frac{1}{3} \\
  \frac{48}{24} & = 2 & \frac{2}{6} & = \frac{1}{3} \\
\end{array}
\]

State whether each sequence is a geometric sequence. If so, state the common ratio and list the next three terms.

1. 1, 2, 4, 8, 16, \cdots  \quad 2. \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}, \cdots 3. \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \cdots

Complete each geometric sequence.

4. 8, 4, 2, \_?, \_?, \_?
5. \_?, 7, 14, 28, \_?, \_?

6. \_?, \frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \_?, \_?
7. \_?, 10^{98}, 10^{96}, \_?, \_?, 10^{90}

Write the first five terms of each geometric sequence described below.

8. first term, 5; ratio, 2  \quad 9. first term, 8; ratio, \frac{1}{2}
10. first term, 81; ratio, \frac{1}{3} \quad 11. first term, \frac{8}{9}; ratio, \frac{3}{2}
A convenient form for writing very large and very small numbers is called **scientific notation**. All numbers expressed in scientific notation are given as the product of a number between 1 and 10 and a power of 10.

Write $2.31 \times 10^{-4}$ in standard form as follows:

$2.31 \times 10^{-4} \rightarrow 0.000231$

To multiply by $10^{-4}$, move the decimal point 4 places to the left.

Write $2.31 \times 10^4$ in standard form as follows:

$2.31 \times 10^4 \rightarrow 23,100$

To multiply by $10^4$, move the decimal point 4 places to the right.

Write 4000 in scientific notation as follows:

$4000 = 4 \times 10^3$

Move the decimal point 3 places to the left. Multiply by $10^3$.

Write 0.00712 in scientific notation as follows:

$0.00712 = 7.12 \times 10^{-3}$

Move the decimal point 3 places to the right. Multiply by $10^{-3}$.

**Write each number in standard form.**

1. $7 \times 10^5$
2. $-3.2 \times 10^3$
3. $1.7 \times 10^{-6}$
4. $6.366 \times 10^{-4}$
5. $2.979 \times 10^4$
6. $-5.09 \times 10^{-7}$

**Write each number in scientific notation.**

7. 50,000
8. -3700
9. 0.000249
10. 2030
11. 0.0000755
12. 51,000
13. 0.0046
14. -12,800
15. -724
16. 0.176
17. -2156
18. 0.03278
To solve problems by **working backward**, start with the end result and *undo* each step.

A certain number is added to 7, and the result is multiplied by 17. The final answer is 136. Find the number.

**Explore**  
The final answer is 136. You want to find the original number.

**Plan**  
Since this problem gives the end result and asks for something that happened earlier, start with the end result and work backward.

**Solve**  
The final answer

\[ \rightarrow 136 \]

*Undo* multiplication by 17.

\[ \rightarrow 136 \div 17 = 8 \]

*Undo* addition of 7.

\[ \rightarrow 8 - 7 = 1 \]

The original number is 1.

**Examine**  
Suppose that you start with the number 1. After adding 7, the result is 8. Multiplying 8 by 17 the result is 136.

---

**Solve by working backward.**

1. A certain number is multiplied by 4, and then 12 is added to the result. The final answer is 36. Find the number.

2. A certain number is divided by 8, and then 4 is subtracted from the result. The final answer is 76. Find the number.

3. Vladimir won some money on a radio contest. He gave half of the money to Julianne and $20 to Hector. Vladimir ended up with $84. How much did he originally win?

4. Pennie received her tax refund. She put $150 of her refund into savings and paid $200 for her car insurance. Then she spent one-half of what was left on a gift and $48 on a sweatshirt. How much was Pennie’s tax refund if she had $56 left?
Some equations contain more than one operation. To solve an equation with more than one operation, use the work-backward strategy and undo each operation.

**Example:** Solve \( \frac{c}{2} - 13 = 7 \).

\[
\frac{c}{2} - 13 + 13 = 7 + 13 \quad \text{Add to undo subtraction.}
\]

\[
\frac{c}{2} = 20
\]

\[
2 \cdot \frac{c}{2} = 20 \cdot 2 \quad \text{Multiply to undo division.}
\]

\[
c = 40
\]

Check the solution.

\[
\frac{c}{2} - 13 = 7 \quad \text{Replace } c \text{ with } 40.
\]

\[
\frac{40}{2} - 13 = 7
\]

\[
20 - 13 = 7
\]

\[
7 = 7 \quad \checkmark \quad \text{The solution is } 40.
\]

**Solve each equation. Check your solution.**

1. \( 4a - 10 = 42 \)
2. \( 12 - 3m = 18 \)
3. \( -10 = -5w - 25 \)
4. \( \frac{m}{4} + 6 = 70 \)
5. \( -3 + \frac{c}{2} = 12 \)
6. \( \frac{-v}{3} + 8 = 22 \)
7. \( 5.8t + 15 = -14 \)
8. \( 8 - 6.2u = -23 \)
9. \( -4 - 2.4w = -16 \)
10. \( 4(x + 6) = 12 \)
11. \( -13 = \frac{4 - b}{3} \)
12. \( -16 = 4(2 - 2x) \)
13. \( -1.4(a + 2) = 4.2 \)
14. \( 7.7 = 2.1 - 7m \)
15. \( \frac{a + 4}{2} = 10.8 \)
Example: The World Trade Center in New York City is 1350 feet tall. Suppose each floor is about 12.3 feet high. You ride the elevator from the top floor downward. How many floors have you passed when you drop to 120 feet?

Explore You know the height of the building and the height of each floor. You are looking for the number of floors passed when you drop to 120 feet.

Plan Let \( f \) = the number of floors. Write an equation.

\[
\frac{1350 \text{ feet}}{12.3 \text{ feet per floor}} = 120
\]

Solve

\[
1350 - 12.3f = 120
\]

\[
-12.3f = 120 - 1350
\]

\[
-12.3f = -1230
\]

\[
f = \frac{1230}{12.3}
\]

Examine After 100 floors, you have dropped 1230 feet. Since 1350 \(-\ 1230\) is 120, the answer is correct.

Define a variable and write an equation for each situation. Then solve.

1. During one day in 1918, the temperature in Granville, North Dakota, began at \(-33^\circ\) and rose for 12 hours. The high temperature was 49.8\(^\circ\). How many degrees per hour did the temperature rise?

2. During one day in 1943, the temperature in Sparkfish, South Dakota, began at \(-4^\circ\) and rose an average of 0.41 degrees per minute until it was 45.2\(^\circ\). How long did this temperature increase take?

3. A skydiver jumps from an airplane at an altitude of 12,000 feet. After 42 seconds, she reaches 11,370 feet and opens her parachute. How many feet per second did she descend before opening her parachute?

4. A skydiver jumps from an airplane at an altitude of 12,775 feet. He descends 15 feet per second. In how many seconds will his altitude be 10,000 feet?
Examples: $C = \text{circumference}; d = \text{diameter}; r = \text{radius}; \text{use } 3.14 \text{ for } \pi$.

$C = \pi d$

$C = \pi d$

$C \approx 3.14(6)$

$C \approx 18.84$

$C \approx 19 \text{ cm}$

$C = 2\pi r$

$d = 2r$

$C = 2\pi r$

$C \approx 2(3.14)(5)$

$C \approx 10(3.14)$

$C \approx 31.4$

$C \approx 31 \text{ m}$

Find the circumference of each circle.

1. 2. 3. 4. The radius is $6 \frac{1}{5}$ feet.

5. The diameter is 4.7 yards.

Solve. Round to the nearest inch.

6. What is the circumference of the top of an ice cream cone if its diameter is about $1 \frac{7}{8}$ inches? ($\frac{7}{8} = 0.875$)

7. The radius of the basketball rim is 9 inches. What is the circumference?
When an equation has the variable on each side, the first step is to write an equivalent equation with the variable on just one side.

\[ 4a - 25 = 6a + 50 \]

\[ 4a - 25 + (-6a) = 6a + 50 + (-6a) \quad \text{Add } -6a \text{ to each side}. \]

\[ -2a - 25 = 50 \]

From this point, the equation is solved by first adding 25 to each side, and then dividing each side by \(-2\).

1. Complete the solution of the equation in the example above.

2. Solve the equation in the example again. This time, first isolate the variable on the right side.

Solve each equation. Check your solution.

3. \[ 4a + 26 = 50 + 6a \]

4. \[ 2r + 36 = 6r - 12 \]

5. \[ 4(b + 24) = 16b + 60 \]

6. \[ x + 21 = -x + 87 \]

7. \[ 25c + 17 = -5c + 143 \]

8. \[ 8v + 5 = 7v - 21 \]

9. \[ 6n - 42 = 4n \]

10. \[ 2x = 3x + 2 \]

11. \[ 5y = 2y - 12 \]

12. \[ a = 5a - 28 \]

13. \[ 2w + 3 = 5w \]

14. \[ -45m + 68 = 84m - 61 \]
The inequality \(13 - 6y > 49 - 2y\) can be solved by applying the same steps as solving equations.

\[
\begin{align*}
13 - 6y &> 49 - 2y \\
13 - 6y - 13 &> 49 - 2y - 13 \\
-6y &> 36 - 2y \\
\text{Subtract 13 from each side} \\
-6y + 2y &> 36 - 2y + 2y \\
-4y &> 36 \\
\text{Add 2y to each side.} \\
\frac{-4y}{-4} &< \frac{36}{-4} \\
y &< -9 \\
\text{Divide each side by -4.} \\
\text{Reverse the order symbol.} \\
\text{Any number less than -9 is a solution.}
\end{align*}
\]

**Check:** Replace \(y\) with a number less than -9. Try -10.

\[
\begin{align*}
13 - 6y &> 49 - 2y \\
13 - 6(-10) &> 49 - 2(-10) \\
13 + 60 &> 49 + 20 \\
73 &> 69 \checkmark
\end{align*}
\]

**Solve each inequality and check your solution. Graph the solution on the number line.**

1. \(6x + 14 \leq 32\)

2. \(18 - 3w < 15\)

3. \(5n + 8 > 20 - n\)

4. \(2y + 2 \geq 6y + 6\)

5. \(\frac{1}{2}(10h - 6) > -23\)

6. \(4(7 - 3x) \leq -20\)

7. \(-15 \leq \frac{c}{-4} - 15\)

8. \(\frac{-n}{3} - 5 < -4\)
You purchase three notebooks at 65¢ each and some pens at 35¢ each. How many pens can you buy if you have $3?

Explore You want to find the amount of pens.

Plan Let \( n \) represent the amount. Write and solve an inequality.

\[
\begin{align*}
\text{Solve} & \quad \text{three times notebook cost plus pen cost times pen amount is at most } \$3 \\
& \quad \frac{3(65)}{35} + \frac{35n}{35} \leq 300 \\
& \quad 195 + 35n \leq 300 \\
& \quad 195 + 35n - 195 \leq 300 - 195 \\
& \quad \frac{35n}{35} \leq \frac{105}{35} \\
& \quad n \leq 3 \\
\end{align*}
\]

Examine Suppose you buy 2 pens, a number less than 3.
The total cost will be \( 3(65\text{¢}) + 2(35\text{¢}) \) or \$2.65.
\$2.65 \leq \$3.00, so you will have enough money.

Define a variable and write an inequality for each situation. Then solve.

1. Twice a number decreased by six is at least 18. What is the number?

2. The sum of a positive even integer and the next greater even integer is at most 14. What are the integers?

3. Four times a number decreased by nine times the same number is greater than 35. What is the number?

4. Alicia spent at least $23.50 on calico and plaid fabric. She bought 5 yards of calico at $1.99 per yard. What is the most she spent on the plaid fabric?
To convert units within the metric system, multiply or divide by powers of ten.

Larger units to smaller units: MULTIPLY →

Unit of length

\[ \text{KILO-} \quad \text{hecto-} \quad \text{deka-} \quad \text{BASE} \quad \text{deci-} \quad \text{CENTI-} \quad \text{MILLI-} \]

\[ \div 10 \quad \div 10 \quad \div 10 \quad \times 10 \quad \times 10 \quad \times 100 \]

Smaller units to larger units: ←DIVIDE

**Examples:**

- \(2.3 \text{ mm} = \underline{?} \text{ cm}\)
  - Smaller to larger means fewer units. Divide by 10.
  - \(2.3 \div 10 = 0.23\)
  - \(2.3 \text{ mm} = 0.23 \text{ cm}\)

- \(6 \text{ kg} = \underline{?} \text{ g}\)
  - Larger to smaller means more units. Multiply by 1000.
  - \(6 \times 1000 = 6000\)
  - \(6 \text{ kg} = 6000 \text{ g}\)

- \(35 \text{ mL} = \underline{?} \text{ L}\)
  - Smaller to larger means fewer units. Divide by 1000.
  - \(35 \div 1000 = 0.035\)
  - \(35 \text{ mL} = 0.035 \text{ L}\)

**Complete each sentence.**

1. \(2.5 \text{ m} = \underline{?} \text{ cm}\)
2. \(0.35 \text{ m} = \underline{?} \text{ mm}\)
3. \(565 \text{ m} = \underline{?} \text{ km}\)
4. \(17 \text{ cm} = \underline{?} \text{ m}\)
5. \(0.3 \text{ cm} = \underline{?} \text{ mm}\)
6. \(2100 \text{ cm} = \underline{?} \text{ km}\)
7. \(3.4 \text{ km} = \underline{?} \text{ m}\)
8. \(53 \text{ cm} = \underline{?} \text{ m}\)
9. \(2 \text{ m} = \underline{?} \text{ cm}\)
10. \(800 \text{ m} = \underline{?} \text{ km}\)
11. \(42 \text{ mm} = \underline{?} \text{ cm}\)
12. \(4600 \text{ mL} = \underline{?} \text{ L}\)
13. \(8 \text{ L} = \underline{?} \text{ mL}\)
14. \(786 \text{ cm} = \underline{?} \text{ m}\)
15. \(3571 \text{ mg} = \underline{?} \text{ g}\)
16. \(3 \text{ kg} = \underline{?} \text{ g}\)
17. \(58 \text{ g} = \underline{?} \text{ mg}\)
18. \(0.045 \text{ m} = \underline{?} \text{ mm}\)
A relation is a set of ordered pairs. The set of first coordinates is called the **domain**. The second set of coordinates is called the **range**.

\[ \{(0, 1), (3, 4), (5, 6)\} \]

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-1</td>
<td>3</td>
</tr>
</tbody>
</table>

The domain is \{0, 3, 5\}. The range is \{1, 4, 6\}.

A **function** is a relation in which each element of the domain is paired with exactly one element in the range.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>2</th>
<th>-6</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

This relation is a function. \{(4, 1), (-2, 6), (4, 3), (1, 0)\}

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>

The domain is \{2, 6\}. The range is \{-1, 3\}.

Write the domain and range of each relation.

1. \{(0, 7), (5, 3), (3, 7), (2, 7)\}

   - The domain is \{0, 3, 5\}.
   - The range is \{1, 4, 6\}.

   - The domain is \{1, 4\}.
   - The range is \{-1, 3, 7\}.

Express the relation shown in each table or graph as a set of ordered pairs. Then state the domain and range of the relation.

1. \((x, y)\) table

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>-4</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>

   - Each element of the domain is paired with exactly one element in the range.

   - The element 4 in the domain is paired with two elements, 3 and 1, in the range.

2. Graph

   - The domain is \{-\frac{1}{4}, \frac{1}{3}, \frac{2}{3}, -10\}, \{-\frac{6}{2}, 42\}.

   - The domain is \{-1, 0, -1, 4\}.

   - The range is \{-1, 3, 0\}.

   - The range is \{2, 4\}.

Determine whether each relation is a function.

6. Graph

   - This relation is not a function.

   - This relation is a function.

   - Determine whether each relation is a function.

7. \{(8, 7), (4, -2), (0, 0), (8, 7)\}

   - Each element of the domain is paired with exactly one element in the range.

   - The element 4 in the domain is paired with two elements, 3 and 1, in the range.

8. \((x, y)\) table

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>5</td>
</tr>
</tbody>
</table>
A scatter plot is a graph of ordered pairs that shows the relationship between two sets of data.

If the points on a graph seem to slant upward to the right, there is a positive relationship.

If the points appear to slant downward to the right, there is a negative relationship.

If the points seem random, there is no relationship.

Example: The scatter plot on the right shows the relationship between the heights of a group of people and their shoe sizes. In this graph, the points go upward to the right. Therefore, there is a positive relationship. In general, the scatter plot seems to show that the taller a person is, the greater the shoe size.

What type of relationship, positive, negative, or none, is shown by each scatter plot?

1. [Graph of Number of Telephone Calls per Week vs. Number in Family]
2. [Graph of Heartbeats per Minute vs. Age in Years]
3. [Graph of Weight lost in 2 Months vs. Hours of Exercise per Week]

Determine whether a scatter plot of the data for the following might show a positive, a negative, or no relationship. Explain your answer.

4. study time, higher grades
5. height, intelligence
6. shoe size, salary
7. age of car, value of car
8. miles per gallon, gas expense
9. education, salary
10. wrist circumference, appetite
11. birthdate, ring size
12. windchill, ice cream sales
13. age of tree, number of rings
14. amount of snowfall, shovel sales
15. hair length, hat size
To find a solution of an equation that has two variables, choose any value for \( x \), substitute that value into the equation and find the corresponding value for \( y \).

\[
\begin{align*}
\text{Suppose } x &= 4 \\
\text{Replace } x & \text{ with } 4.
\end{align*}
\]

\[
y = 7 + 4 \\
y = 11
\]

When \( x = 4 \), \( y = 11 \). So the ordered pair \((4, 11)\) is a solution of the equation \( y = 7 + x \).

An equation has many ordered pairs that are solutions. For example, four ordered pairs for the equation \( y = x - 1 \) are \((3, 2), (0, -1), (2, 1), \) and \((-1, -2)\). To graph an equation, graph the ordered pairs and then draw the line that contains the points. An equation is called a **linear equation** if its graph is a straight line.

**Find four solutions for each equation. Write the solutions as ordered pairs.**

1. \( y = x \)
2. \( y = x + 1 \)
3. \( y = x - 2 \)
4. \( 2x - y = 3 \)
5. \( y = -4x + 1 \)
6. \( y = \frac{1}{2}x + 3 \)

**Determine whether each relation is linear.**

7. \( -3x + 2y = 1 \)
8. \( y = x^2 - 1 \)
9. \( y = -x - 3 \)

**Graph each equation.**

10. \( y = 2x - 4 \)
11. \( -3x + y = -5 \)
12. \( y = -\frac{1}{2}x \)
Suppose \( y = 3x + 2 \) and the domain is \( \{-2, 0, 1\} \). Make a table of the domain and corresponding range values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Using the vertical line test, we find that \( y = 3x + 2 \) is a function.

Equations that represent functions can be written in **functional notation**, \( f(x) \). The symbol \( f(x) \) represents the value in the range that corresponds to the value of \( x \) in the domain. The equation \( y = 3x + 2 \) can be written as \( f(x) = 3x + 2 \).

To determine a functional value, substitute the given value for \( x \) in the equation. For example, if \( f(x) = 3x + 2 \) and \( x = -3 \), then \( f(-3) = 3(-3) + 2 \) or -7.

---

**For each equation,**

a. solve for the domain = \( \{-4, 0, 1\} \), and

b. determine whether the equation is a function.

1. \( x + y = 12 \)
2. \( x = 3 - y \)
3. \( y = 8 + x^2 \)

---

**Given** \( h(x) = 2x - 9 \) and \( g(x) = x^2 + 4 \), **find each value.**

4. \( h(-3) \)
5. \( g(-5) \)
6. \( 5[h(0)] \)
7. \( g\left(\frac{-1}{4}\right) \)
8. \( g(2.2b) \)
9. \( g(3a) \)
10. \( h(-0.48) \)
11. \( 2[g(2)] \)
12. \( h\left(\frac{1}{2}\right) \)
The interest Leon earned on $160 was $8. If he had deposited $200, he would have earned $10. How much money would he need to deposit to earn $12?

**Explore**  Given the interest earned for $160 and $200, you need to determine how much money to deposit to earn $12.

**Plan**  Graph the given information. Then read the graph to find how much money to deposit to earn $12.

**Solve**  Let the horizontal axis represent the amount deposited. Let the vertical axis represent the amount earned. Graph the ordered pairs $(160, 8)$ and $(200, 10)$. Draw a line that contains these points. He will need to deposit $240.

**Examine**  Recall that Leon earned $8 for $160. Since $12 = 8 \times 1.5$, the deposit should be $1.5 \times 160$ or $240$ to earn $12$ in interest.

**Use a graph to solve each problem. Assume that the rate is constant in each problem.**

1. Scoopers ice cream shop sells one scoop of ice cream for $1.60. They sell three scoops for $4.80. How much money is needed to buy two scoops of ice cream?

2. Kevin earned $90 for baby-sitting 15 hours. He would have earned $120 for five more hours. How much does he charge per hour? How much will Kevin earn if he works 10 hours?

3. Sami measures the heights of the steps going into her house. The 2nd step is 1 foot above ground. The 5th step is $2\frac{1}{2}$ feet above ground. What is the height of the 11th step?
The steepness of a line is called its **slope**. The vertical change is called the **change in y**, and the horizontal change is called the **change in x**.

\[
\text{slope} = \frac{\text{change in } y}{\text{change in } x}
\]

**Example:** In the graph above, the change in y is 2, and the change in x is 3. Therefore, the slope of the line is \(\frac{2}{3}\).

The slope of a line can also be found by using the coordinates of any two points on the line.

\[
\text{slope} = \frac{\text{difference in } y}{\text{difference in } x} \quad \text{or} \quad \frac{\text{difference in } y\text{-coordinates}}{\text{difference in } x\text{-coordinates}}
\]

**Example:** Find the slope of the line that contains the points \(A(-1, -2)\) and \(B(-4, -3)\).

\[
slope = \frac{-2 - (-3)}{-1 - (-4)} = \frac{1}{3}
\]

Find the slope of each line.

1. ![Graph 1](image)
2. ![Graph 2](image)
3. ![Graph 3](image)

Find the slope of the line that contains each pair of points.

4. \(R(-2, -3), S(-1, -1)\)  
5. \(T(-4, -2), U(-2, -1)\)  
6. \(V(-4, 1), W(2, 0)\)  
7. \(P(1, -2), Q(-5, -2)\)  
8. \(L(1, 4), M(1, -3)\)  
9. \(M(-2, -4), N(-1, -1)\)
The point where a graph intersects an axis is called an **intercept** of the graph.

The lines at the right cross both axes. Line \(a\) crosses they \(y\)-axis at \((0, 2)\). Therefore, the \(y\)-intercept is 2. Notice that the \(x\) value of the \(y\)-intercept is 0.

The **\(y\)-intercept** is the value of an equation when \(x = 0\). The **\(x\)-intercept** is the value when \(y = 0\). To graph a linear equation using the \(x\)- and \(y\)-intercepts, find the intercepts. Graph them and then draw the line that contains them.

**Example:** Graph \(y = x + 3\) using the \(x\)- and \(y\)-intercepts.

To find the \(x\)-intercept, let \(y = 0\). 
\[ y = x + 3 \]
\[ 0 = x + 3 \]
\[ -3 = x \]
The \(x\)-intercept is -3.
The ordered pair is \((-3, 0)\).

To find the \(y\)-intercept, let \(x = 0\). 
\[ y = x + 3 \]
\[ y = 0 + 3 \]
\[ y = 3 \]
The \(y\)-intercept is 3.
The ordered pair is \((0, 3)\).

**State the \(x\)-intercept and the \(y\)-intercept for each line.**

1. \(a\)
2. \(b\)
3. \(c\)
4. \(d\)
5. \(e\)
6. \(f\)

**Use the \(x\)-intercept and \(y\)-intercept to graph each equation.**

7. \(y = 1 - 2x\)
8. \(y = \frac{1}{2}x + 1\)
9. \(-x - 3y = 3\)
The equations \( y = \frac{1}{2}x + 2 \) and \( y = 3x - 5 \) together are called a system of equations.

The solution to this system is the ordered pair that is the solution of both equations. To solve a system of equations, graph each equation on the same coordinate plane. The point where both graphs intersect is the solution of the system of equations.

Line \( l \) is the graph of \( y = \frac{1}{2}x + 2 \).

Line \( n \) is the graph of \( y = 3x - 5 \).

The lines intersect at (2, 1).
Therefore, the solution to the system of equations is (2, 1).

**Use a graph to solve each system of equations.**

1. \( y = x - 3 \)  
   \( y = -3x + 1 \)

2. \( y = 2x \)  
   \( y = x + 1 \)

3. \( y = 4x + 5 \)  
   \( y = 4x - 1 \)

4. \( y = x \)  
   \( y = -2 \)

5. \( y = -x - 1 \)  
   \( y = -3x - 3 \)

6. \( y = 6x - 12 \)  
   \( y = 2x - 4 \)
1. Graph the following ordered pairs on the coordinate system at the right.

(2, 3)  (3, 5)  (-2, -1)
(-3, 2)  (3, 4)  (-1, 0)

2. Where do these points lie in the plane in relation to the graph of \( y = x \)?

3. In each ordered pair in Exercise 1, is the \( x \)-coordinate less than, equal to, or greater than the \( y \)-coordinate?

4. Which of the following do the ordered pairs in Exercise 1 represent: \( y = x \), \( y > x \), or \( y < x \)?

5. Which of the following represents the points located below the graph of \( y = x \): \( y = x \), \( y > x \), or \( y < x \)?

6. To represent all ordered pairs \((x, y)\) where \( y > x \), shade the portion of the coordinate plane above the graph of \( y = x \). Note that the dashed line means that the graph of \( y = x \) is not part of the graph of \( y > x \).

7. Which of the following belong to the graph of \( y > x \)?

   (10, 20)  \( (\frac{1}{4}, \frac{1}{2}) \)  \( (\frac{1}{2}, \frac{1}{4}) \)  (0, 0)

---

**Graph each inequality.**

8. \( y < 3x \)

9. \( y > 2x + 3 \)

10. \( x < 1 - y \)
A **ratio** is a comparison of two numbers by division. If the two terms of a ratio have no common factors, the ratio is in simplest form.

\[
\frac{4}{16} = \frac{1}{4}
\]

The GCF of 4 and 16 is 4.

One type of ratio is a **rate**. A rate compares two measurements with different units. Speeds, such as 50 miles per hour or 32 feet per second, are familiar examples of rates. To change a rate to a unit rate, divide both the numerator and denominator by the denominator.

\[
\text{Rate} \div 4 = \text{Unit Rate} \div 4
\]

**Write the ratio that compares each of the following.**

1. number of p’s to number of i’s in *Mississippi*
2. number of o’s to total number of letters in proportion
3. number of months that have an r in their name to the number of months in a year

**Express each ratio or rate as a fraction in simplest form.**

4. 9 to 12
5. 12 to 9
6. 5:20
7. $2.50 for 5 notepads
8. 60¢ per dozen
9. $3.00 to rent 2 videotapes

**Express each ratio as a unit rate.**

10. 120 miles in 2 hours
11. 800 pounds for 40 square inches
12. $300 for 5 jackets
13. 45 meters in 3 minutes
14. 10 kilometers in 2 hours
15. 30 yards in 15 seconds
Example: Every week Evelyn and her sister Maria save some of their allowance to buy a present for their mother. Evelyn can save $0.90 a week and Maria can save $0.75. How many weeks will it take them to save at least $11.00?

Explore It would help in solving this problem to think of a way to keep track of how much money they have saved.

Plan One way to keep track is to use a table.

Solve

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evelyn</td>
<td>$0.90</td>
<td>$1.80</td>
</tr>
<tr>
<td>Maria</td>
<td>$0.75</td>
<td>$1.50</td>
</tr>
</tbody>
</table>

1. Fill in the table above until you have found the solution to the problem in the example. Be sure to examine your solution.

2. Henrietta and Samuel start out driving separately from the same house on their way to their uncle’s house. Henrietta drives 55 miles each hour and Samuel drives 50 miles each hour. How far will Samuel have traveled when Henrietta gets to her uncle’s house, which is 385 miles away?

| Henrietta | 55 |
| Samuel | 50 |

3. Mario’s dartboard has three scoring rings on it. The outer ring is only worth 1 point, the middle ring is worth 3 points, and the inner ring is worth 7 points. If he places the dart in the inner ring only once, and hits the outer ring twice as often as the middle ring, how many points has he scored in each of the three rings if his final score is 42 points? How many darts has he thrown for this score?

| 7 |
| 3 |
| 1 |
In mathematics, the study of chance is called **probability**.

The **probability** of an event is defined as the number of ways a certain outcome can occur divided by the number of possible outcomes.

When tossing a coin, there are two outcomes—heads or tails. Each outcome is equally likely. In other words, the probability of heads appearing is the same as that of tails appearing.

An outcome that definitely will happen has a probability of 1. An outcome that cannot happen has a probability of 0.

---

**A cooler contains 2 cans of grape juice, 3 cans of grapefruit juice, and 7 cans of orange juice. A can of juice is chosen without looking. Find each probability.**

1. \( P(\text{grapefruit juice}) \)  
2. \( P(\text{orange juice}) \)  
3. \( P(\text{grape juice}) \)

**A die is rolled. Find each probability.**

4. \( P(\text{5}) \)  
5. \( P(\text{2}) \)  
6. \( P(\text{2, 4, or 6}) \)

7. \( P(\text{3 or 4}) \)  
8. \( P(\text{not 6}) \)  
9. \( P(\text{1}) \)

**There are 4 grape, 2 cherry, 3 lemon, and 7 raspberry gumballs in a bag. Suppose you select one gumball at random. Find each probability.**

10. \( P(\text{cherry gumball}) \)  
11. \( P(\text{lemon or grape gumball}) \)

12. \( P(\text{raspberry or grape gumball}) \)  
13. \( P(\text{lemon gumball}) \)

14. \( P(\text{peppermint gumball}) \)  
15. \( P(\text{cherry or spearmint gumball}) \)

16. \( P(\text{grape, cherry, lemon, or raspberry gumball}) \)
A proportion is a statement of equality of two or more ratios. To determine if two ratios form a proportion, check their cross products. If the cross products are equal, the ratios form a proportion.

\[
\frac{1}{2} = \frac{2}{4} \quad \frac{2}{5} = \frac{6}{15} \quad \frac{2}{3} = \frac{10}{12}
\]

\[
\frac{1}{2} \times 2 = \frac{2}{4} \quad \frac{2}{5} \times 6 = \frac{12}{15} \quad \frac{2}{3} \times 10 = \frac{20}{15}
\]

1 \times 4 \neq 2 \times 2 
2 \times 15 \neq 5 \times 6 
2 \times 12 \neq 3 \times 10

\begin{align*}
4 = 4 & \quad \text{It is a proportion.} \\
30 = 30 & \quad \text{It is a proportion.} \\
24 \neq 30 & \quad \text{It is not a proportion.}
\end{align*}

Cross products can be used to solve proportions.

**Example:** Solve the proportion \( \frac{3}{4} = \frac{x}{20} \).

\[
3 \cdot 20 = 4 \cdot x \\
60 = 4x \\
15 = x
\]

Multiply.

Divide each side by 4.

Write \( = \) or \( \neq \) in each blank to make a true statement.

1. \( \frac{3}{8} \quad \frac{12}{32} \)
2. \( \frac{15}{20} \quad \frac{3}{4} \)
3. \( \frac{4}{7} \quad \frac{16}{49} \)

4. \( \frac{1}{2} \quad \frac{1}{4} \)
5. \( \frac{35}{50} \quad \frac{7}{10} \)
6. \( \frac{40}{48} \quad \frac{5}{6} \)

**Solve each proportion.**

7. \( \frac{5}{8} = \frac{x}{40} \)
8. \( \frac{6}{3} = \frac{10}{t} \)
9. \( \frac{n}{5} = \frac{42}{7} \)

10. \( \frac{4}{11} = \frac{12}{x} \)
11. \( \frac{2}{3} = \frac{0.8}{n} \)
12. \( \frac{7}{12} = \frac{1.68}{b} \)

**Write a proportion that could be used to solve each problem. Then solve the proportion.**

13. Cole can pick 2 rows of beans in 30 minutes. How long will it take him to pick 5 rows if he works at the same rate?

14. A tree casts a shadow 30 meters long. A 2.8-meter pole casts a shadow 2 meters long. How tall is the tree?
The proportion shown at the right is called the percent proportion. It can be used to solve problems involving percent.

The percentage \( (P) \) is a number that is compared to another number called the base \( (B) \). The rate is a percent. Always compare \( r \) to 100.

**Example:** Of the 800 tomatoes in a crop, 60% will be used to make ketchup. How many tomatoes will be made into ketchup?

Use the proportion \( \frac{P}{B} = \frac{r}{100} \).

\[
\frac{P}{800} = \frac{60}{100}
\]

\[P \cdot 100 = 800 \cdot 60\]

\[100P = 48,000\]

\[P = 480\]

There will be 480 tomatoes made into ketchup.

**Use the percent proportion to solve each problem.**

1. Find 70% of 90.
2. Find 15% of 400.

3. What number is 75% of 600?
4. What number is 50% of 96?

5. 20% of 140 is what number?
6. 45% of 32 is what number?

7. Find 60% of 60.
8. Find 24% of 10.5.

9. 100% of 8.73 is what number?
10. What number is 98% of 230?

11. Joan's income is $190 per week. She saves 20% of her weekly salary. How much does she save each week?
12. Ninety percent of the seats of a flight are filled. There are 240 seats. How many seats are filled?
Mr. Niles takes a poll of 10 students in his class. Of the 10 students polled, 3 prefer to have the test today, and 7 prefer to have the test tomorrow. The 10 students polled are a sample of all the students in the class. The result of the poll can be used to predict the number of students who prefer to have the test tomorrow.

There are 30 students in Mr. Niles’s class. How many of these students would you expect to prefer to take the test tomorrow?

Explore

What is given? 10 students polled; 3 prefer to have the test today and 7 prefer tomorrow.

What is asked? 30 students in the class; how many prefer the test tomorrow?

Plan

Assume that the sample is representative of the entire class. Set up a proportion to show two equivalent ratios. Let $t$ represent the total number of students that prefer to have the test tomorrow.

Solve

$$\frac{t}{10} = \frac{7}{30}$$

Solve for $t$.

$$7 \times 30 = 10t$$
$$210 = 10t$$
$$21 = t$$

The solution is 21.

Mr. Niles predicts that 21 students would prefer to have the test tomorrow.

Examine

Replacing $t$ in the original equation with 21, we see that $\frac{7}{10} = \frac{21}{30}$. Therefore, the answer is correct.

Solve. Use the poll shown below.

<table>
<thead>
<tr>
<th>How many days per week should you have physical education?</th>
<th>13</th>
<th>20</th>
<th>27</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>one day</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>two days</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>three days</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>four days</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. How many people are in the sample?
2. What part of the sample chose two days?
3. What percentage of the sample chose four days?
4. What part of the sample chose three days?
5. Suppose there are 240 people in the school. How many do you predict would say three days?
Fractions, decimals, and percents can all be used to represent the same number.

**Example:** Express 2.45 as a mixed number and as a percent.

<table>
<thead>
<tr>
<th>mixed number</th>
<th>percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.45 → (\frac{9}{20}) → (2\frac{9}{20})</td>
<td>2.45 → 2.45 → 245%</td>
</tr>
</tbody>
</table>

**Example:** Express \(\frac{1}{4}\) as a decimal and as a percent.

<table>
<thead>
<tr>
<th>decimal</th>
<th>percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{4}) → 0.25</td>
<td>(\frac{1}{4} = \frac{r}{100}) (100 = 4r) (25 = r) (\frac{1}{4} = 25%)</td>
</tr>
</tbody>
</table>

**Express each percent or fraction as a decimal.**

1. 49%  
2. 185%  
3. 16.9%  
4. \(\frac{2}{5}\)  
5. \(\frac{21}{40}\)  
6. \(\frac{5}{8}\)  
7. \(1\frac{1}{2}\)  
8. 4%

**Express each decimal or fraction as a percent.**

9. 5.62  
10. 0.327  
11. 0.007  
12. \(\frac{25}{100}\)  
13. \(\frac{5}{6}\)  
14. \(3\frac{2}{5}\)  
15. 0.6  
16. \(2\frac{3}{10}\)

**Express each percent or decimal as a fraction or a mixed number.**

17. 45%  
18. 150%  
19. 0.3  
20. 0.235  
21. 4.5  
22. 0.55  
23. 0.005  
24. 56%
**9-8 Study Guide**  
Percent and Estimation

<table>
<thead>
<tr>
<th>Actual</th>
<th>Rounded</th>
<th>Fractional Equivalent</th>
<th>Estimate will be a little . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>52%</td>
<td>50%</td>
<td>( \frac{1}{2} )</td>
<td>less</td>
</tr>
<tr>
<td>78%</td>
<td>80%</td>
<td>( \frac{4}{5} )</td>
<td>more</td>
</tr>
</tbody>
</table>

Complete the table below. Use the chart on the right to find the closest percent.

<table>
<thead>
<tr>
<th>Actual</th>
<th>Rounded</th>
<th>Fractional Equivalent</th>
<th>Estimate will be a little . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>24%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>67%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>89%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>76%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>62%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>58%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
To solve percent problems, use the percent equation, \( P = R \cdot B \).

\( R \) represents \( \frac{r}{100} \).

**Example 1:** Find 45% of 36.

\[
P = \text{45\% of 36} = 0.45 \times 36 = 16.2
\]

45% of 36 is 16.2.

**Example 2:** 8 is what percent of 16?

\[
8 = R \times 16
\]

\[
\frac{8}{16} = \frac{16R}{16}
\]

\[
0.5 = R
\]

8 is 50% of 16.

The amount of interest \((I)\) earned on an account depends upon the principal \((p)\), which is the money deposited, the rate \((r)\), and the time \((t)\) given in years.

\[
I = p \times r \times t
\]

Solve each problem by using the percent equation, \( P = R \cdot B \).

1. Find 40% of 80.
2. Find 15% of 600.
3. What number is 30% of 120?
4. What number is 90% of 50?
5. What percent of 250 is 25?
6. What percent of 35 is 7?
7. 100% of 67 is what number?
8. 200% of 67 is what number?

Find the interest to the nearest cent.

9. $160 at 5.5\% for 1.25 years
10. $1800 at 6.5\% for 2 years
11. $350 at 6\% for 6 months
12. $7050 at 6\% for 3 months
13. $3500 at 10\% for 5 years
14. $75 at 12\% for 6 years
Example: Al’s Sporting Goods Store raised the price of one of its best-selling bicycles from $125 to $140. Find the percent of increase.

Subtract to find the amount of change. \(140 - 125 = 15\)
Solve the percent equation. Compare the amount of increase to the original amount.
\[
P = R \cdot B
\]
\[
15 = R \cdot 125
\]
\[
\frac{15}{125} = \frac{125R}{125}
\]
\[
0.12 = R
\]
The percent of change is 12%.

Example: Al’s Sporting Goods Store reduced the price of one of its less-popular bicycles from $80 to $60. Find the percent of decrease.

Divide the new amount by the original amount. \(60 \div 80 = 0.75\)
Subtract 1 from the result and write the decimal as a percent. \(0.75 - 1 = -0.25\) or -25%
The percent of change is -25%. The percent of decrease is 25%.

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of increase or decrease. Round to the nearest whole percent.

1. a $120 turntable now costs $150
2. a $6 album is now $4.20
3. a $100 digital watch is now $72
4. a $32 sweater now costs $36.80

5. Cheryl weighed 120 pounds. Shedieted and lost 12 pounds in three weeks. Find the percent of change in Cheryl's weight.
6. The junior high school's enrollment changed from 1200 to 1350 students. Find the percent of change in enrollment.

7. Potatoes baked in the oven require 60 minutes to cook. A pressure cooker can do the same job in 20 minutes. Find the percent of change in cooking time.
8. A $320 stereo amplifier is on sale for a limited time at $264. Find the percent of change in price.
The diagram at the right called a **stem-and-leaf plot**. It is one way to organize a list of numbers. This plot shows some of the numbers between 0 and 40.

The key indicates that $3\mid 4$ represents 34. So, the “stems” are the tens-place digits, and the “leaves” are the ones-place digits. The numbers shown by the plot are listed below.

3, 5, 12, 12, 17, 18, 20, 21, 26, 26, 29, 34, 35, 35, 35

### Write the numbers shown by each stem-and-leaf plot.

1. \[0\mid 1 \ 4 \ 6 \ 6 \\
   1 \ 0 \ 3 \ 4 \ 9 \ 9 \\
   2 \ 2 \ 3 \ 3 \ 3 \ 4 \ 8 \\
   3 \ 0 \ 3 \ 5 \ 5 \\
   0\mid 4 = 4\]

2. \[5\mid 0 \ 0 \ 2 \ 4 \ 8 \\
   6 \ 1 \ 3 \ 6 \\
   7 \ 2 \ 5 \ 5 \ 8 \\
   8 \ 3 \ 7 \ 8 \ 8 \ 9 \ 9 \\
   6\mid 1 = 61\]

3. Use the stem-and-leaf plot in Exercise 1.
   a. How many numbers are shown on the plot?
   b. Which number(s) appears most frequently?
   c. Which numbers appear least frequently?

   a. Which number(s) appear most frequently?
   b. Are there more numbers greater than 70, or less than 70?
   c. How many numbers are shown?

5. The following numbers are the results of a survey. A group of ninth-grade students were asked to report the number of hours they spent watching television in one week. Complete the stem-and-leaf plot at the right, using the results of the survey.

0, 12, 25, 19, 23, 7, 7, 5, 26, 16,

28, 0, 1, 0, 12, 25, 10, 2, 25, 18,

23, 1, 14, 0, 26, 19, 14, 21, 25
The list below shows the test scores for a sample of 11 high school students.

\[ 65 \quad 65 \quad 67 \quad 72 \quad 75 \quad 75 \quad 80 \quad 87 \quad 92 \quad 93 \]

The difference between the least and the greatest number in the set is called the **range**. The range above is 93–65 or 28.

When a list of data is arranged in order, the **median** is the middle number. The median above is enclosed in a box.

The circled numbers in the list above are used to analyze the data. The **lower quartile** is the median of the lower half of the data. The **upper quartile** is the median of the upper half.

The difference between the lower quartile and the upper quartile is called the **interquartile range**. The interquartile range above is 87–67 or 20.

Recall that when there are two middle numbers, the median is their mean. This is also true for the upper and lower quartiles.

---

**Find the range, median, upper and lower quartiles, and the interquartile range for each set of data.**

1. 48, 50, 53, 50, 44, 52, 45

2. 32, 0, 6, 20, 0, 12, 15, 25, 18, 15, 24

3. **Ages of Eastside Health Club Members**

<table>
<thead>
<tr>
<th>Name</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aimee</td>
<td>19</td>
</tr>
<tr>
<td>Lucille</td>
<td>35</td>
</tr>
<tr>
<td>Leonard</td>
<td>20</td>
</tr>
<tr>
<td>Tiago</td>
<td>52</td>
</tr>
<tr>
<td>Nakeisha</td>
<td>43</td>
</tr>
<tr>
<td>Hector</td>
<td>27</td>
</tr>
<tr>
<td>Robin</td>
<td>44</td>
</tr>
<tr>
<td>Haleem</td>
<td>20</td>
</tr>
<tr>
<td>Marrissa</td>
<td>32</td>
</tr>
<tr>
<td>Tad</td>
<td>27</td>
</tr>
</tbody>
</table>

4. **Movies That Jerome Attended Each Year**

<table>
<thead>
<tr>
<th>Year</th>
<th>Movies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987</td>
<td>22</td>
</tr>
<tr>
<td>1988</td>
<td>12</td>
</tr>
<tr>
<td>1989</td>
<td>27</td>
</tr>
<tr>
<td>1990</td>
<td>34</td>
</tr>
<tr>
<td>1991</td>
<td>3</td>
</tr>
<tr>
<td>1992</td>
<td>18</td>
</tr>
<tr>
<td>1993</td>
<td>40</td>
</tr>
<tr>
<td>1994</td>
<td>14</td>
</tr>
<tr>
<td>1995</td>
<td>10</td>
</tr>
</tbody>
</table>
George surveyed a group of students who walk to school. This list shows the number of minutes it takes each person to walk from his or her house to the school.

5 5 10 15 20 25 30 30 30 30 30 35 40 45 50

The median is in the box. The lower and upper quartiles are circled.

A **box-and-whisker plot** is shown at the right. A box is drawn to show the median (30), lower quartile (15), and upper quartile (35). Other key numbers from the data list are the lower extreme (5) and upper extreme (50).

**Use the box-and-whisker plots to answer each question below them.**

1. 
   - a. What is the median?
   - b. What are the lower quartile and the upper quartile?
   - c. What are the lower and upper extremes?

2. 
   - a. What is the median?
   - b. What are the lower quartile and the upper quartile?
   - c. What are the lower and upper extremes?

3. Make a box-and-whisker plot at the right for this list of test scores.
   60, 65, 65, 70, 75, 75, 80, 85, 85, 85, 85, 85, 85, 85, 90

4. Make a box-and-whisker plot at the right for this list of data. It shows the number of pets per family.
   0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 5
The way in which data is displayed in a graph can cause the graph to be visually misleading. The two graphs below show the number of cookies sold each month by a gourmet cookie shop.

The graphs above contain the same data. However, the graph on the right suggests lower sales than the graph on the left. This is due to the shortened vertical axis. Also, notice that the vertical axis does not include zero.

**The graph at the right displays data about money earned by high school students during summer vacation.**

1. Is the graph misleading? Explain.

2. Kimiko made two different graphs showing employee productivity levels for each year in business.

   **Productivity Level - Graph A**

   2. Which graph is misleading? Why?

3. If Kimiko wanted to ask the management to give the employees raises in 1996, which graph would be used? Explain your answer.
Example: Cheryl has a choice of a pink, red, or yellow blouse with white or black slacks for an outfit. How many possible outfits are there?

Draw a tree diagram to determine the number of different outfits.

Blouses     Slacks
pink        white → PW
            black → PB
red         white → RW
            black → RB
yellow      white → YW
            black → YB

There are 6 possible outfits.

You can also find the number of possible outcomes by multiplying.

<table>
<thead>
<tr>
<th>Number of different blouses</th>
<th>Number of different slacks</th>
<th>Total possible outfits</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

**Draw a tree diagram to find the number of outcomes for each situation.**

1. A bag contains one red marble and one white marble. A second bag contains two red marbles and a white one. How many different outcomes are possible if a marble is picked from each bag? The marbles are replaced each time.

2. How many different lunches can be made with a choice of hot dog, salad, or chili; pop, milk, or lemonade; and crackers, bread, or a roll?

**Find the number of possible outcomes for each event.**

3. At an ice cream shop, there are 31 flavors and 25 toppings. How many different ways are there to make a one-scoop ice cream sundae with one topping?

4. There are 4 quarterbacks and 6 centers on a football team that has 60 players. How many quarterback-center pairings are possible?
An arrangement where order is important is called a permutation.

Example: Mario, Sandy, Fred, and Shana are running for the offices of president, secretary, and treasurer. In how many ways can these offices be filled?

Any of the 4 people can fill the president’s position. Once the president has been chosen, there are 3 people to fill the secretary’s position. That leaves 2 people to fill the treasurer’s position.

The symbol \( P(4, 3) \) represents the number of permutations of 4 things taken 3 at a time.

\[
P(4, 3) = 4 \times 3 \times 2, \text{ or } 24.
\]

The offices can be filled 24 ways.

An arrangement where order is not important is called combination.

Example: Charles has four coins in his pocket and pulls out three at a time. How many different amounts can he get? The coins are a penny, a nickel, a dime, and a quarter.

There are \( 4 \times 3 \times 2, \) or 24 different outcomes, but some are the same. To find the number of different combinations, divide the number of permutations \( P(4, 3), \) by the number of different ways three items can be arranged.

\[
C(4, 3) = \frac{P(4, 3)}{3!} = \frac{4 \times 3 \times 2}{3 \times 2 \times 1} \text{ or } 4.
\]

4 different amounts can be chosen.

\[^{3!} \text{ is read “three factorial.”}\]

Find each value.

1. \( 5! \)
2. \( P(4, 2) \)
3. \( 8! \)
4. \( C(15, 6) \)
5. \( P(3, 3) \)
6. \( 13! \)
7. \( C(5, 5) \)
8. \( P(10, 4) \)

9. In how many ways can five books be arranged on a shelf? (Order is important.)
10. In how many ways can three students council members be elected from five candidates? (Order is not
The odds in favor of an event is the ratio of the number of ways the outcome can occur to the number of ways the outcome cannot occur.

**Example:** Samantha has 2 quarters, 5 dimes, 4 nickels, and 10 pennies in her bank. If one coin is chosen, what are the odds that it is a penny or a quarter?

Odds of a penny choose ways to choose penny or a quarter or quarter other coins

\[
\text{Odds of a penny} = \frac{\text{ways to choose penny}}{\text{ways to or quarter other coins}} = \frac{5}{12} : \frac{9}{12} = 5 : 9 = 4 : 3
\]

This is read “4 to 3.”

**Find the odds of each outcome if a card is drawn from the cards at the right.**

1. an even number

2. a number less than 6

3. not 2 or 7

4. odd or even

5. 7 or a multiple of 2

6. a number greater than 6

**Find the odds of each outcome if a laundry bag contains 3 dress shirts, 6 dish towels, 8 socks, 2 pairs of jeans, and 5 T-shirts.**

7. a dish towel

8. a pair of jeans or a sock

9. not a T-shirt

10. *neither* dress shirt *nor* sock

11. sock, dish towel, or dress shirt

12. dish towel or T-shirt
There are three gumball machines. Each machine contains an equal number of red, blue, and yellow gumballs. If Robin gets one gumball from each machine, what is the probability that two of the gumballs are red?

**Explore**  
There are three gumball machines. There are three different colors of gumballs available. We need to find the probability that two out of three gumballs will be red.

**Plan**  
Since three colors are available in equal amounts, a spinner like the one at the right can be used to simulate the situation. Spin the spinner and record the results. Repeat the simulation ten times.

<table>
<thead>
<tr>
<th>Gumball Machine</th>
<th>Number 1</th>
<th>Number 2</th>
<th>Number 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation #1</td>
<td>Y</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>Simulation #2</td>
<td>Y</td>
<td>B</td>
<td>Y</td>
</tr>
<tr>
<td>Simulation #3</td>
<td>Y</td>
<td>B</td>
<td>Y</td>
</tr>
<tr>
<td>Simulation #4</td>
<td>B</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Simulation #5</td>
<td>R</td>
<td>B</td>
<td>R</td>
</tr>
<tr>
<td>Simulation #6</td>
<td>R</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>Simulation #7</td>
<td>Y</td>
<td>Y</td>
<td>R</td>
</tr>
<tr>
<td>Simulation #8</td>
<td>B</td>
<td>Y</td>
<td>B</td>
</tr>
<tr>
<td>Simulation #9</td>
<td>Y</td>
<td>B</td>
<td>Y</td>
</tr>
<tr>
<td>Simulation #10</td>
<td>Y</td>
<td>R</td>
<td>B</td>
</tr>
</tbody>
</table>

One of the simulations results in two red gumballs. You can estimate the probability of getting two red gumballs will be $\frac{1}{10}$.

**Examine**  
The actual probability that two of the three gumballs are red is $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$. The estimate is reasonable.

1. The aquarium at the pet store contains an equal amount of goldfish that have either orange or white fins. Linda chooses six fish at random. What is the probability that three of the six fish have white fins?

2. The M & W Bakery makes homemade white, wheat, rye, pumpernickel, garlic, and Italian bread. The baker chooses a type of bread at random to sell each day. What is the probability that the baker sells the same type of bread four days in one week?

3. What is the probability that a family of four children has three girls and one boy?
If the outcome of one event does not influence the outcome of a second event, the events are independent.

**Example:** A jar contains 12 red bells and 12 silver bells. Pick one, replace it, and pick another. The probability of picking a silver bell twice is

\[ \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \]

If the outcome of the second event depends on the outcome of the first event, the events are dependent.

**Example:** A jar contains 12 red bells and 12 silver bells. Pick one, keep it and pick another. The probability of picking two red bells is

\[ \frac{1}{2} \times \frac{11}{23} = \frac{11}{46} \]

Refer to the ten buttons on the left to find the probability of each outcome. Each button is replaced.

1. a white button twice  
2. a gray button twice  
3. a gray button, then a white button  
4. a white button, then a black button  
5. a black button twice  
6. a black button, then a gray button

Refer to the ten buttons shown above to find the probability of each outcome. Each button is not replaced.

7. a white button twice  
8. a gray button twice  
9. a gray button, then a white button  
10. a white button, then a black button  
11. a black button twice  
12. a black button, then a gray button
When two events cannot happen at the same time, they are mutually exclusive. For two events that are mutually exclusive, \( P(A \text{ or } B) = P(A) + P(B) \).

**Example:** A die is tossed. Find \( P(5 \text{ or } 6) \).

\[
P(5 \text{ or } 6) = P(5) + P(6) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}
\]

When two events can occur at the same time, they are inclusive. For two events that are inclusive, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \).

**Example:** A die is tossed. Find \( P(\text{even or greater than } 4) \).

\[
P(\text{even or } > 4) = P(\text{even}) + P(> 4) - P(\text{even and } > 4) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}
\]

**Determine whether each event is mutually exclusive or inclusive. The find the probability.**

1. The drawing at the right shows the 18 best seats at a concert hall. The tickets for these seats are given away during a raffle.

   a. a seat in row A or row C

   b. a seat in row A or a seat numbered 3

   c. a seat numbered 2 or 5

   d. a seat in row A, B, or C

   e. a seat numbered 1, 2, or 6

   f. an aisle seat or a seat in row C

   g. a seat in row C or a prime-numbered seat

   h. an odd-numbered seat or a seat in row C

   i. a seat not in row B or a seat numbered 2
The point indicates a specific location. A straight path of points that extends infinitely in two directions is called a line. A plane is a flat surface with no boundaries, or edges. A line segment consists of two endpoints and all the points between them. A ray is part of a line that has one endpoint and extends from one point indefinitely in one direction. An angle is formed by two rays with a common endpoint called the vertex. Angles are measured in degrees.

A right angle has a measure of 90°.

An acute angle has a measure between 0° and 90°.

An obtuse angle has a measure between 90° and 180°.

Use the figure at the right to name an example of each term.

1. ray
2. point
3. angle
4. line
5. line segment
6. vertex

Classify each angle as acute, right, or obtuse.

7. \( \angle FBH \)
8. \( \angle CBD \)
9. \( \angle ABC \)
10. \( \angle ABG \)
11. \( \angle ABE \)
12. \( \angle EBH \)
13. \( \angle DBH \)
14. \( \angle FBG \)
A circle graph is used to illustrate data. In order to make a circle graph, the circle must be separated into sectors. Each circle is made up of 360°. Therefore, to separate the circle into sectors, multiply the percent needed by 360.

Example: 25% of a family’s budget is spent for housing.

\[0.25 \times 360° = 90°\]

Use a protractor and measure 90°. Then label the sector “Housing” as shown below.

1. The chart shows the percentages allotted for each item of the Johnson’s family budget. Make a circle graph to display the data.

<table>
<thead>
<tr>
<th>Family Budget</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing</td>
<td>25%</td>
</tr>
<tr>
<td>Savings</td>
<td>15%</td>
</tr>
<tr>
<td>Food</td>
<td>27%</td>
</tr>
<tr>
<td>Transportation</td>
<td>10%</td>
</tr>
<tr>
<td>Medical</td>
<td>10%</td>
</tr>
<tr>
<td>Other</td>
<td>13%</td>
</tr>
</tbody>
</table>

![Johnson Family Budget](image)

2. In a recent poll, a group of people were asked to name their favorite type of television show. Make a circle graph to display the data.

<table>
<thead>
<tr>
<th>TV Preference</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Movies</td>
<td>12%</td>
</tr>
<tr>
<td>Sports</td>
<td>20%</td>
</tr>
<tr>
<td>News</td>
<td>4%</td>
</tr>
<tr>
<td>Drama</td>
<td>16%</td>
</tr>
<tr>
<td>Comedy</td>
<td>20%</td>
</tr>
<tr>
<td>Music</td>
<td>28%</td>
</tr>
</tbody>
</table>

![Favorite TV Shows](image)
Opposite angles formed by intersecting lines are called vertical angles. Vertical angles are always congruent. \(\angle 1\) and \(\angle 3\), and \(\angle 2\) and \(\angle 4\) are pairs of vertical angles.

The sum of the measures of two complementary angles is \(90^\circ\).

The sum of the measures of two supplementary angles is \(180^\circ\).

Line \(n\) is called a transversal. When two parallel lines, \(\ell\) and \(m\), are intersected by a transversal, certain pairs of angles are congruent.

**Corresponding angles** are congruent. 
Pairs of corresponding angles are 2 and 6, 4 and 8, 1 and 5, and 3 and 7.

**Alternate interior angles** are congruent. 
Pairs of alternate interior angles are 3 and 6, and 4 and 5.

**Alternate exterior angles** are congruent. 
Pairs of alternate exterior angles are 1 and 8, and 2 and 7.

Each pair of angles is either complementary or supplementary. 
Find the value of \(x\) in each figure.

1. 
2. 
3. 

In the figure at the right, \(r \parallel s\). If the measure of \(\angle 4\) is \(38^\circ\), find the measure of each angle.

4. \(\angle 1\) 
5. \(\angle 7\) 
6. \(\angle 2\) 
7. \(\angle 6\) 
8. \(\angle 8\) 
9. \(\angle 3\) 
10. \(\angle 5\)
A right triangle has one right angle. An obtuse triangle has one obtuse angle. All other triangles are acute. In an acute triangle, each of the three angles is acute.

For any triangle, the sum of the measures of the angles is 180°.

**Example:** In \( \triangle AOD \), the measure of \( \angle D \) is 35°, and the measure of \( \angle O \) is 107°. Find the measure of \( \angle A \).

\[
\begin{align*}
m\angle A + m\angle O + m\angle D &= 180 \\
m\angle A + 107 + 35 &= 180 \\
m\angle A + 142 &= 180 \\
m\angle A &= 38
\end{align*}
\]

The sum of the measures of the angles is 180.
Replace \( m\angle O \) with 107 and \( m\angle D \) with 35.
Add 107 and 35.
Subtract 142 from each side.

**Find the value of** \( x \). **Then classify each triangle as acute, right, or obtuse.**

1. \( \triangle \) with angles 83°, 32°, and \( x \)°.
2. \( \triangle \) with angles 45°, 45°, and \( x \)°.
3. \( \triangle \) with angles 58°, 63°, and \( x \)°.
4. \( \triangle \) with angles 35°, 35°, and \( x \)°.

**Use the figure at the right to solve each of the following.**

5. Find \( m\angle 2 \) if \( m\angle 3 = 49° \) and \( m\angle 1 = 63° \).

6. Find \( m\angle 2 \) if \( m\angle 3 = 45° \) and \( m\angle 1 = 50° \).

7. Find \( m\angle 3 \) if \( m\angle 2 = 66° \) and \( m\angle 1 = 64° \).

8. Find \( m\angle 3 \) if \( m\angle 2 = 38° \) and \( m\angle 1 = 70° \).

9. Find \( m\angle 3 \) if \( m\angle 2 = 42° \) and \( m\angle 1 = 58° \).
The triangles on the right are congruent. The parts of congruent triangles that “match” are called corresponding parts.

\[ \angle A \text{ corresponds to } \angle G. \]  
\[ \overline{AB} \text{ corresponds to } \overline{GH}. \]

\[ \angle B \text{ corresponds to } \angle H. \]  
\[ \overline{BC} \text{ corresponds to } \overline{HF}. \]

\[ \angle C \text{ corresponds to } \angle F. \]  
\[ \overline{AC} \text{ corresponds to } \overline{GF}. \]

**Complete the congruence statement for each pair of congruent triangles. Then name the corresponding parts.**

1. \( \triangle TRS \cong \triangle \) ______

2. \( \triangle ABD \cong \triangle \) ______

3. \( \triangle FHG \cong \triangle \) ______
Triangles that have the same shape but not necessarily the same size are similar. In similar triangles, the measures of corresponding angles are equal, and the measures of corresponding sides are proportional.

When some measures of the sides of similar triangles are unknown, proportions can be used to find the missing measures.

Example: If \( \triangle ABC \sim \triangle DEF \), find the value of \( x \).

\[
\frac{\text{width of } \triangle ABC}{\text{width of } \triangle DEF} = \frac{30}{12} = \frac{20}{x} \quad \text{height of } \triangle ABC \quad \text{height of } \triangle DEF
\]

\[30 \cdot x = 12 \cdot 20 \]
\[30x = 240 \]
\[x = 8 \]

The height of \( \triangle DEF \) is 8 feet.

Write a proportion to find each missing measure \( x \). Then find the value of \( x \).

1. \( \frac{24\text{ in.}}{x\text{ in.}} = \frac{8\text{ in.}}{6\text{ in.}} \)

2. \( \frac{x\text{ km}}{24\text{ km}} = \frac{5\text{ km}}{12\text{ km}} \)

3. \( \frac{7\text{ mm}}{x\text{ mm}} = \frac{5\text{ mm}}{10\text{ mm}} \)

4. \( \frac{18\text{ m}}{x\text{ m}} = \frac{30\text{ m}}{12\text{ m}} \)

5. A basketball pole is 10 feet high and casts a shadow of 12 feet. A girl standing nearby is 5 feet tall. How long is the shadow that she casts?

6. Use similar triangles to find the distance across the pond.
You can classify quadrilaterals by their pairs of parallel sides.

For any quadrilateral, the sum of the measures of the angles is 360°.

**Find the value of x.**

1. 91° + 133° + 89° + x° = 360°
2. 86° + 118° + x° = 360°
3. x° = 360° - 42° - 137°

**Find the value of x. Then find the missing angle measures.**

4. x° + 125° = 180°
5. x° + 122° = 180°
6. (x - 30)° + (x - 30)° + 90° + 3x° = 360°

**Classify each quadrilateral using the name that best describes it.**

7. Parallelogram
8. Trapezoid
9. Rectangle
Polygons are simple, closed figures formed by three or more line segments, called sides. If a polygon has $n$ sides, then the sum of the measures of the interior angles is $(n - 2)180$.

**Example:** Find the sum of the measures of the angles of a heptagon.

A heptagon has 7 sides. Therefore, $n = 7$.

\[(n - 2)180 = (7 - 2)180 = 5(180) \text{ or } 900\]

The sum of the measures of the angles of a heptagon is $900^\circ$.

Regular polygons are figures in which all sides are congruent and all angles are congruent. Since the heptagon above is regular, the measure of one interior angle is $900 \div 7$, or about $129^\circ$.

Interior and exterior angles of a polygon are supplementary. In a regular heptagon, the measure of the interior angle is about $129^\circ$. Therefore, the measure of the exterior angle is $180 - 129$, or about $51^\circ$.

### Find the sum of the measures of the interior angles of each polygon.

1. hexagon  
2. pentagon  
3. quadrilateral

4. octagon  
5. 16-gon  
6. 27-gon

### Find the measure of each exterior angle and each interior angle of each regular polygon.

7. regular octagon  
8. regular decagon  
9. regular 12-gon

10. regular heptagon  
11. regular 15-gon  
12. regular 24-gon

### Find the perimeter of each regular polygon.

13. regular heptagon with sides 12 ft long  
14. regular quadrilateral with sides 3.7 m long

15. regular octagon with sides $\frac{1}{2}$ yd long  
16. regular pentagon with sides $2\frac{4}{5}$ in. long
Transformations are movements of geometric figures. When a geometric figure is moved horizontally, vertically, or both, it is called a **translation**. The figure at the right is moved 6 units to the left.

In a **rotation**, a figure is turned about a point.

When a figure is “flipped” over a line, it is called a **reflection**. At the right, \( \triangle ABC \) is reflected about line \( \ell \). Since the figure can be folded over line \( \ell \) so that the two halves correspond, the figure is **symmetric**. Line \( \ell \) is called a line of symmetry. A line of symmetry separates a figure into two congruent parts.

The figure at the right has three lines of symmetry.

**Tell whether each transformation is a translation, a rotation, or a reflection. Explain your answer.**

1. 

2. 

3. 

**Draw all lines of symmetry.**

4. 

5. 

6. 

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To find the area of a parallelogram, multiply the base by the height.

**Example:** Find the area of the parallelogram.

\[ A = b \times h \]

\[ = 5 \times 3 \]

\[ = 15 \text{ ft}^2 \]

The area of a triangle is half the product of the base and the height.

**Example:** Find the area of the triangle.

\[ A = \frac{1}{2} \times b \times h \]

\[ = \frac{1}{2} \times 6 \times 4 \]

\[ = 12 \text{ yd}^2 \]

To find the area of a trapezoid, multiply the sum of the bases by one-half the height.

**Example:** Find the area of the trapezoid.

\[ A = \frac{1}{2} \times h \times (a + b) \]

\[ = \frac{1}{2} \times 7 \times (12 + 22) \]

\[ = \frac{1}{2} \times 238 \]

\[ = 119 \text{ km}^2 \]

**Find the area of each figure.**

1. 2. 3. 4. 5. 6.
The area of a circle with radius \( r \) can be found by using the formula below.

\[
A = \pi r^2
\]

Find the area of the circle at the left.

\[
A = \pi (4)^2 = \pi \cdot 16 \approx 50.27 \text{ cm}^2
\]

Find the area of each circle. Round to the nearest tenth.

1. \( 1\frac{1}{2} \) in.
2. 27 mm
3. 3 ft
4. 15 cm
5. \( 8\frac{3}{4} \) yd
6. 6.1 m
7. diameter, 22 yd, \( 380.1 \) yd\(^2\)
8. radius, 8.4 in., \( 221.7 \) in\(^2\)
9. radius \( 2\frac{1}{4} \) in., \( 15.9 \) in\(^2\)
10. diameter, 6.5 ft, \( 33.2 \) ft\(^2\)
11. diameter, 4 cm, \( 12.6 \) cm\(^2\)
12. radius, 4.6 mm, \( 66.5 \) mm\(^2\)
Geometric probability involves using area to find the probability of an event.

**Example:** The figure at the right represents a dartboard. Assume that a dart will land on the board and that the dart is equally likely to land any place on the board. Find the probability of landing in the shaded region.

\[
P(\text{shaded}) = \frac{\text{shaded area}}{\text{area of target}}
\]

First find the area of the target and the area of the shaded region. Then write a fraction to represent the probability of the dart landing in the shaded region.

The dartboard is a 7-foot by 7-foot square.
\[A = s^2\]
\[= 7^2\]
\[= 49\]

The dartboard is 49 square feet.

Then shaded region is 3 feet by 3 feet.
\[A = s^2\]
\[= 3^2\]
\[= 9\]

The shaded region is 9 square feet.

So, \[P(\text{dart landing in shaded region}) = \frac{9}{49} \approx 0.184\]

The probability of landing in the shaded region is \[\frac{9}{49}\] or about 18%.

Each figure represents a dartboard. Find the probability of landing in the shaded region.

1. 
2. 
3. 
4. 
5. 
6.
A rectangular playground is 65 feet long and 35 feet wide. A sidewalk is placed around the playground. If the sidewalk extends 4 feet around the playground, what is the area of the sidewalk?

**Explore** You know the dimensions of the playground and the width of the sidewalk. You need to find the area of the sidewalk.

**Plan** Make a drawing to illustrate the problem. You can find the area of the sidewalk by subtracting the area of the smaller section from the area of the larger section.

**Solve**

Area of the larger section: \[ A = \ell \cdot w \]
\[ = (65 + 4 + 4) \cdot (35 + 4 + 4) \]
\[ = 73 \cdot 43 \]
\[ = 3139 \text{ ft}^2 \]

Area of the smaller section: \[ A = \ell \cdot w \]
\[ = 65 \cdot 35 \]
\[ = 2275 \text{ ft}^2 \]

The area of the sidewalk is \(3139 - 2275\), or 864 square feet.

**Examine** An answer of 864 square feet is reasonable, compared to the area of the playground, 2275 square feet.

---

1. A 15-foot-by-12-foot room is to be carpeted. There is a 3-foot-by-5-foot area in front of the fireplace where one-foot-square tiles are to be installed. How many square feet of carpet is needed?

2. The Warren’s backyard is 50-foot-by-50-foot square. They have attached an 8-foot chain to a back corner fence post to tie up their dog. How much of an exercise area does the dog have?

3. The Keiko Construction Company is building a house. They need to dig a rectangular basement. It will have a length of 32 feet, a width of 24 feet, and a depth of 8 feet. How much dirt must be removed?

4. A wall of the basement is 32 feet by 8 feet. The wall is to be constructed with masonry blocks that have lengths of 16 inches and widths of 8 inches. About how many masonry blocks are needed to build the wall?
To find the surface area of a solid, find the area of each surface. Then add the areas.

**Rectangular Prism**

Area:
- **Top and bottom:** \(2 \cdot (5 \cdot 3) = 30 \text{ cm}^2\)
- **Front and back:** \(2 \cdot (2 \cdot 5) = 20 \text{ cm}^2\)
- **Two sides:** \(2 \cdot (2 \cdot 3) = 12 \text{ cm}^2\)

Total surface area \(= 62 \text{ cm}^2\)

**Cylinder**

Area:
- **Top and bottom:** \(2 \cdot \pi \cdot (7)^2 \approx 307.7 \text{ in}^2\)
- **Curved surface:** \(2 \cdot \pi \cdot 7 \cdot 14 \approx 615.8 \text{ in}^2\)

Total surface area \(\approx 923.5 \text{ in}^2\)

Find the surface area of each solid. Round to the nearest tenth.

1. Rectangular Prism

2. Cylinder

3. Cylinder

4. Rectangular Prism

5. Rectangular Prism

6. Triangular Prism
To find the surface area of a pyramid, add the areas of each of the faces and the area of the base.

To find the surface area of a cone, add the area of its circular base and the area of its lateral surface. The area of the lateral surface is $\pi r\ell$.

Area:

**Square Base**

$A = s^2$

$= 6^2$

$= 36 \text{ in}^2$

**Four Triangular faces**

$A = 4\left(\frac{1}{2} \cdot b \cdot h\right)$

$= 4\left(\frac{1}{2} \cdot 6 \cdot 8\right)$

$= 96 \text{ in}^2$

Surface area is $36 + 96$ or $132 \text{ in}^2$.

Area:

**Circular Base**

$A = \pi r^2$

$\approx (3.14)(3)^2$

$\approx 28.26 \text{ in}^2$

**Lateral Surface**

$A = \pi r\ell$

$\approx (3.14)(3)(10)$

$\approx 94.2 \text{ in}^2$

Surface area is about $28.26 + 94.2$ or $122.46 \text{ in}^2$.

*Find the surface area of each pyramid or cone. Round to the nearest tenth.*

1. 2.

3.

4.

5.

6.
The volume \((V)\) of an object is the amount of space that a solid contains. Volume is measured in cubic units.

To find the volume of any prism, multiply the area of the base \((B)\) by the height \((h)\).

**Example:**
Find the volume of the triangular prism.

\[
V = Bh \\
= \left(\frac{1}{2} \cdot 10 \cdot 12\right) \cdot 15 \\
= 60 \cdot 15 \\
= 900
\]

The volume is 900 m\(^3\).

The base of a circular cylinder is a circle. Therefore, substitute \(\pi r^2\) for \(B\).

**Example:**
Find the volume of the cylinder.

\[
V = Bh \\
= \pi r^2 h \\
= (\pi)(2)^2(8) \\
= \pi \cdot 32 \\
\approx 100.5
\]

The volume is about 100.5 cm\(^3\).

**Find the volume of each prism or cylinder. Round to the nearest tenth.**

1. \(6\text{ in.} \times 6\text{ in.}\)\(6\text{ in.}\)
2. \(4\text{ m} \times 9\text{ m}\)
3. \(4\text{ cm} \times 8\text{ cm} \times 3\text{ cm}\)
4. \(9\text{ cm} \times 12\text{ cm}\)
5. \(0.3\text{ km} \times 0.6\text{ km} \times 0.8\text{ km}\)
6. \(11\text{ cm} \times 4\text{ cm}\)
The volume of a pyramid is one-third the volume of a prism with the same base and height as the pyramid.

\[ V = \frac{1}{3} B \times h \]

\[ = \frac{1}{3} \times \ell \times w \times h \quad B = \ell \times w \]

\[ = \frac{1}{3} \times 10 \times 11 \times 18 \]

\[ = 660 \text{ The volume is } 660 \text{ cm}^3. \]

The volume of a cone is one-third the volume of a cylinder with the same radius and height as the cone.

\[ V = \frac{1}{3} \pi r^2 h \]

\[ = \frac{1}{3} \times 3.14 \times 3 \times 3 \times 12 \]

\[ = 113.04 \text{ The volume is about } 113.04 \text{ m}^3. \]

Use the drawings above.

1. The prism has a height of 18 cm and a base that is 11 cm by 10 cm. Find its volume.

2. The cylinder has a height of 12 m and a base with a radius of 3 m. Find its volume. Use \( \pi = 3.14 \).

Find the volume of each pyramid or cone. Round to the nearest tenth.

3. 

4. 

5. 

6. 

7. 

8.
Since \(6 \times 6 = 36\), a square root of 36 is 6.

\[\sqrt{36} = 6\]

Since \(-6 \times (-6) = 36\), \(-6\) is also a square root of 36. A negative sign is used to indicate the negative square root.

\[-\sqrt{36} = -6\]

If the square root of a number is a whole number, the original number is called a perfect square. For example, 81 is a perfect square because \(9 \times 9 = 81\). However, 79 and 80 are not perfect squares.

In cases where the square root of a number is not a whole number, you can estimate the square root by using the two closest perfect squares. Seventy is between 64 and 81. So, \(\sqrt{70}\) is between \(\sqrt{64}\) and \(\sqrt{81}\) or between 8 and 9. Since 70 is closer to 64 than to 81, \(\sqrt{70}\) is closer to 8.

Find each square root.

1. \(\sqrt{9}\)  
2. \(\sqrt{25}\)  
3. \(\sqrt{4}\)  
4. \(-\sqrt{64}\)

5. \(\sqrt{121}\)  
6. \(-\sqrt{196}\)  
7. \(\sqrt{225}\)  
8. \(\sqrt{1.44}\)

9. \(\sqrt{900}\)  
10. \(-\sqrt{324}\)  
11. \(\sqrt{529}\)  
12. \(\sqrt{100}\)

Find the best integer estimate for each square root. Then check your estimate using a calculator.

13. \(-\sqrt{37}\)  
14. \(\sqrt{90}\)  
15. \(-\sqrt{50}\)

16. \(\sqrt{300}\)  
17. \(-\sqrt{75}\)  
18. \(\sqrt{69}\)

19. \(\sqrt{1681}\)  
20. \(\sqrt{27.96}\)  
21. \(-\sqrt{11.25}\)
At Calera City High School, 180 students participate in the band and/or a sport. One hundred thirty-four students are in the band, and 90 play sports. Forty-four students participate in both activities. How many students participate in sports only?

**Explore** You know the total amount of students, how many students are in the band, how many play sports, and how many participate in both activities.

**Plan** Draw a Venn diagram to organize the data.

**Solve** Draw two intersecting circles in a rectangle to represent sports and the band. Write the number of students who participate in both activities. Then subtract to find the number of students who participate only in one activity.

- Band = 134 - 44 = 90
- Sports = 90 - 44 = 46

There are 46 students that participate in sports only.

**Examine** Look at the Venn diagram. Add the number of students in each region.

90 + 44 + 46 = 180 Since the total is 180, the answer is correct.

**Solve using a Venn diagram.**

1. A television station conducted a survey. They asked 100 viewers which Star Trek series they prefer, The Next Generation or Voyager. Sixty-six people preferred The Next Generation, 50 people liked Voyager, and 16 people preferred both equally. How many people liked The Next Generation only?

2. Three hundred fourteen structural engineers were surveyed. One hundred ninety engineers create steel designs, and 216 create wood designs. Ninety-two engineers do both steel and wood designs. How many structural engineers create steel designs but not wood designs?

3. The Venn diagram below represents students’ sports preferences.

   - How many students prefer only football?

4. The Venn diagram below represents students’ subject preferences.

   - How many students like all three subjects?
Rational numbers are numbers that can be expressed as a quotient of two integers, where the divisor is not zero.

Irrational numbers are numbers that can be named by nonterminating, non-repeating decimals.

The set of real numbers includes both the rational numbers and the irrational numbers.

Some equations have solutions that are irrational numbers. To solve an equation that involves squares, take the square root of each side.

Example: Solve \( x^2 = 65 \)

\[
x^2 = 65 \\
x = \sqrt{65} \text{ or } x = -\sqrt{65} \\
x \approx 8.1 \text{ or } x \approx -8.1
\]

Take the square root of each side.

Name the sets of numbers to which each number belongs: the whole numbers, the integers, the rational numbers, the irrational numbers, and/or the reals.

1. 6.01001 . . .
2. -2
3. \(-\frac{3}{5}\)

4. 0.83
5. \(\sqrt{19}\)
6. 0.625

7. \(-\sqrt{81}\)
8. 4.2342352 . . .
9. \(-\sqrt{7}\)

Solve each equation. Round decimal answers to the nearest tenth.

10. \(c^2 = 49\)
11. \(x^2 = 4\)
12. \(w^2 = 14\)

13. \(n^2 = 289\)
14. \(m^2 = 132\)
15. \(h^2 = 250\)

16. \(k^2 = 3.24\)
17. \(r^2 = 1600\)
18. \(d^2 = 90\)
In a right triangle, the square of the hypotenuse, \( c \), is equal to the sum of the squares of the lengths of the other two sides, \( a \) and \( b \).

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
5^2 &= 3^2 + 4^2 \\
5^2 &= 9 + 16 \\
25 &= 25
\end{align*}
\]

**Example:** How long must a ladder be to reach a window 13 feet above ground? For the sake of stability, the ladder must be placed 5 feet away from the base of the wall.

\[
c^2 = (13)^2 + (5)^2 \\
c^2 = 169 + 25 \\
c^2 = 194 \\
c^2 = \sqrt{194} \approx 13.9 \text{ ft}
\]

**Solve. Round decimal answers to the nearest tenth.**

1. In a softball game, how far must the catcher throw to second base?

2. How long must the brace be on a closet rod holder if the vertical side is 17 cm and the horizontal side must be attached 30 cm from the wall?

3. If Briny is 32 miles due east of Oxford and Myers is 21 miles due south of Oxford, how far is the shortest distance from Myers to Briny?

4. In a baseball game, how far must the shortstop (halfway between second base and third base) throw to make an out at first base?
In any $30^\circ-60^\circ$ right triangle, the length of the shortest side is one-half the length of the hypotenuse. The side opposite the $60^\circ$ angle is $\sqrt{3}$ times the length of the other leg.

**Example:**
Find the lengths of sides $a$ and $b$ in the triangle below.

![Triangle with sides labeled (a, b, c)](image)

Side $a$ is one-half of side $c$, 8 cm. Therefore, side $a = \frac{1}{2} \cdot 8$, or 4 cm.

Side $b$ is $\sqrt{3}$ times side $a$. Thus, side $b = \sqrt{3} \cdot 4$, or about 6.9 cm.

In any $45^\circ-45^\circ$ right triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

**Example:**
Find the lengths of sides $a$ and $b$ in the triangle below.

![Triangle with sides labeled (a, b, c)](image)

Side $b$ is the same length as side $a$, 5 in. Therefore, side $c = \sqrt{2} \cdot 5$, or about 7.1 in.

Find the lengths of the missing sides in each triangle. Round decimal answers to the nearest tenth.

1. 

   ![Triangle with sides labeled (b, c, 6.9 mm)](image)

2. 

   ![Triangle with sides labeled (a, b, 30°, 4.8 in.)](image)

3. 

   ![Triangle with sides labeled (22.4 m, d)](image)

4. 

   ![Triangle with sides labeled (7 in., g, f)](image)

5. 

   ![Triangle with sides labeled (4 cm, h, g)](image)

6. 

   ![Triangle with sides labeled (k, j, 20 cm)](image)
In the triangle at the right, the measures of the sides are used to find the following trigonometric ratios.

\[
\text{sine of } \angle R = \frac{\text{measure of the leg opposite } \angle R}{\text{measure of the hypotenuse}} = \frac{4}{5} \text{ or } 0.8
\]

\[
\text{cosine of } \angle R = \frac{\text{measure of the leg adjacent to } \angle R}{\text{hypotenuse}} = \frac{3}{5} \text{ or } 0.6
\]

\[
\text{tangent of } \angle R = \frac{\text{measure of the leg opposite } \angle R}{\text{measure of the leg adjacent to } \angle R} = \frac{4}{3} \text{ or about } 1.333
\]

If a trigonometric ratio is known, then the degree measure of an angle can be determined by using a calculator. For example, in the triangle above, \( \sin \angle R = 0.8 \). Therefore, the measure of \( \angle R \) is \( 0.8 \text{ sin}^{-1} \) or about 53.1°.

**For each triangle, find sin A, cos A, and tan A to the nearest thousandth.**

1. \( \triangle ABC \)

2. \( \triangle DAB \)

3. \( \triangle ABC \)

4. \( \triangle CAF \)

**For each triangle, find the measure of the marked acute angle to the nearest degree.**

5. \( \triangle ABC \)

6. \( \triangle ABC \)

7. \( \triangle ABC \)

8. \( \triangle ABC \)
The ski run at Mad River Mountain rises at an angle of $25^\circ$. The length of the run is 500 meters. How high is the run?

$$\sin 25^\circ = \frac{a}{500}$$

$$0.4226 = \frac{a}{500}$$

$$0.4226 \times 500 = \frac{a}{500} \times 500$$

$$211.3 = a$$

The run is approximately 211.3 meters high.

Write an equation that you could use to solve for $x$. Then solve. Round decimal answers to the nearest tenth.

1. $$\sin 36^\circ = \frac{12}{x}$$
2. $$\tan x^\circ = \frac{9}{9}$$
3. $$\sin x^\circ = \frac{8}{3}$$

Use trigonometric ratios to solve each problem.

4. How high is the basketball hoop?

5. How long is the tent pole?

6. What is the angle of the ramp?

7. What is the angle of the ladder?
A **polynomial** is an algebraic expression that contains one or more monomials. A **binomial** is a polynomial with two terms. A **trinomial** is a polynomial with three terms.

A term of a polynomial cannot be a quotient with the variable in the denominator or the square root of a variable.

To find the degree of a monomial, add the exponents on the variables. The monomial $5x^5y^3$ has a degree of $5 + 3$ or $8$.

The degree of a polynomial is the same as that of the term with the highest degree.

**Example:** Find the degree of $y^3 + 2y^3z + 4x$.

- $y^3$ has degree $3$.
- $2y^3z$ has degree $3 + 1$ or $4$.
- $4x$ has degree $1$.

The degree of $y^3 + 2y^3z + 4x$ is $4$.

**State whether each expression is a polynomial. If it is, classify it as a monomial, binomial, or trinomial.**

1. $r^2 - 2$
2. $\frac{-10}{c}$
3. $\frac{w}{2} - 24$
4. $b^2 + 1$
5. $-14m^2n$
6. $4x^2 - 2xy - 4y^2$
7. $\frac{1}{x} - \frac{1}{y} - \frac{1}{z}$
8. $a^2 - b^3 - c^4$
9. $\sqrt{b} - 4ac$

**Find the degree of each polynomial.**

10. $5d^4 + 13a^2b^4c$
11. $7g^2h - 2h$
12. $2x^4 - 6xy^3 + 3x^3z^2$
13. $-36$
14. $12a + 3$
15. $-34r^2s^4t^6$
To add two or more polynomials, look for **like terms**. Like terms are those that have exactly the same variables to the same powers.

**Method 1:**
**Add Vertically**

\[
\begin{align*}
5x + 1 & \quad \text{Align the terms.} \\
+ 4x + 3 & \quad \text{Add.} \\
9x + 4 & \\
\end{align*}
\]

In Method 1, notice that the terms are aligned and added.

**Method 2:**
**Add Horizontally**

\[
\begin{align*}
(3x + 2y + 1) + (5x + 2) & = (3x + 5x) + (2y) + (1 + 2) \\
& = 8x + 2y + 3 \\
\end{align*}
\]

In Method 2, notice that there is nothing to add to the term \(2y\). That is, the second polynomial does not include a term having the variable \(y\).

**Find each sum.**

1. \(\frac{2x + 3}{+ 3x + 9}\)
2. \(\frac{8y - 7}{+ x + 13}\)
3. \(\frac{12y + 7x}{+ 7y - 2x}\)

4. \(\frac{3x^2 - x + 7}{+ 2x^2 + 3x + 3}\)
5. \(\frac{6x + 12}{+ 8y - 2}\)
6. \(\frac{8x}{+ x + 4y + 11}\)

7. \((4y + 2x) + (6y - x)\)
8. \((6x + 2y) + (4x - 8y)\)
9. \((11m - 4n) + 3n\)

10. \((x + y + 8) + (5x + 2y + 3)\)
11. \((3x^2 - 12x + 7) + (3 + 2x - 6x^2)\)

12. \((5y^2 + 3y + 2) + (2y^2 + 9)\)
13. \((4y^2 - y + 6) + (3y - 4)\)
A rational number can be subtracted by adding its **additive inverse** or opposite. You can find the additive inverse of a number by multiplying the number by \(-1\). For example, the additive inverse of 3 is \((-1)3\), or -3. You can also subtract binomials and trinomials by adding their additive inverse.

To find the additive inverse or binomials and trinomials, replace each term with its additive inverse.

<table>
<thead>
<tr>
<th>binomial</th>
<th>additive inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x + 2y)</td>
<td>(x - 2y)</td>
</tr>
<tr>
<td>(2x^2 + 4x)</td>
<td>(-2x^2 - 4x)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>trinomial</th>
<th>additive inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2x^2 - 3x + 5)</td>
<td>(-2x^2 + 3x - 5)</td>
</tr>
<tr>
<td>(-8x + 5y - 7z)</td>
<td>(-8x - 5y + 7z)</td>
</tr>
</tbody>
</table>

**Examples**

a. \(4x^2 + 5xy\)
   \[-6x^2 + 8xy\]
   additve inverse \((-6x^2 + 8xy)\)
   \[
   \begin{align*}
   4x^2 + 5xy \\
   - (6x^2 + 8xy) \\
   2x^2 - 3xy
   \end{align*}
   \]

b. \(12a^2 - 3a + 3\)
   \[-6a^2 - 12a - 4\]
   additve inverse \((-6a^2 - 12a - 4)\)
   \[
   \begin{align*}
   12a^2 - 3a + 3 \\
   - (6a^2 + 12a + 4) \\
   6a^2 + 9a + 7
   \end{align*}
   \]

**Find each difference.**

1. \((2x + 6) - (x + 4)\)
2. \((6x - 1) - (2x + 5)\)
3. \((14y + 2) - (6y - 1)\)
4. \(9x + y + 8\)
   \[(-)5x + 3y - 3\]
5. \(6x + 7\)
   \[(-) -x - 4y - 12\]
6. \(x^2 + 2x + 4\)
   \[(-) x^2 - 6x + 3\]
7. \((5y^2 + 3y + 2) - (2y^2 - 9)\)
8. \((4y^2 - y + 6) - (-3y - 5)\)
9. \((9x + 12) - (5y + 3)\)
To find a **power of a power**, multiply the exponents.

\[
(4^2)^3 = 4^{2 \cdot 3} = 4^6 \quad \quad (x^3)^4 = x^{3 \cdot 4} = x^{12}
\]

To find a **power of a product**, multiply the individual powers.

\[
(ab)^4 = a^4b^4 \quad \quad (xy)^{-3} = x^{-3}y^{-3}
\]

Use the previous rules to find a **power of a monomial**.

\[
(2a^4b^5)^2 = (2)^2(a^4)^2(b^5)^2 = 2^2a^{4 \cdot 2}b^{5 \cdot 2} = 4a^8b^{10}
\]

\[
(d^2e^6f^3)^{-3} = (d^2)^{-3}(e^6)^{-3}(f^3)^{-3} = d^{-6}e^{-18}f^9
\]

**Simplify.**

1. \((ab)^7\)  
2. \((2^4)^2\)  
3. \((6m)^3\)

4. \((-4m^2n)^2\)  
5. \((k^6)^3\)  
6. \([(-2)^4]^2\)

7. \((-3y^5)^2\)  
8. \((-h^4)^5\)  
9. \((d^6e^3)^8\)

10. \((4a^2b)^4\)  
11. \((-pq^2)^9\)  
12. \(6x(4x)^3\)

13. \((-2x^4y^3)^6\)  
14. \(-4t(t^4)^5\)  
15. \([4(2)^3]^3\)

16. \((-w^8)^{-4}\)  
17. \((-m^9)^{-2}\)  
18. \(7(u^6v)^{-4}\)

**Evaluate each expression if** \(x = -1\) **and** \(y = 4\).

19. \(3x^2y\)  
20. \(-2xy^3\)  
21. \((x^3y)^3\)

22. \(2x(2y^2)^{-2}\)  
23. \((x^2y)^4\)  
24. \(-(xy)^{-2}\)
To multiply a polynomial by a monomial, use the distributive property. First, multiply each term of the polynomial by the monomial. Then combine any like terms.

**Example:** Find the product of \(4x^2y (3x - 2xy^3 + 4y^2 + 2x)\).

\[
4x^2y (3x - 2xy^3 + 4y^2 + 2x) \\
= 4x^2y(3x) - 4x^2y(2xy^3) = 4x^2y(4y^2) + 4x^2y(2x) \quad \text{Distributive property} \\
= 12x^3y - 8x^3y^4 + 16x^2y^3 + 8x^2y \quad \text{Multiplying monomials} \\
= 20x^3y - 8x^3y^4 + 16x^2y^3 \quad \text{Combine like terms.}
\]

**Find each product.**

1. \(2(3x - 4)\) 
2. \(-3z(z + 9)\) 
3. \(4(x - 3)\) 
4. \(7w(3w + 2)\) 
5. \(-2c(c + 7)\) 
6. \(x(y^2 - z)\) 
7. \(3p(p^3 + p^2)\) 
8. \(xy(xy - 3x)\) 
9. \(5y^3(2xy - 3y^2)\) 
10. \(-xy(xy + x^2 - 3)\) 
11. \(2x(xy + x^2 - 3)\) 
12. \(-xy(2xy - 3y^2)\) 
13. \(2x(2xy - 3y^2 + 4)\) 
14. \(5(-2x^2 + 3x - 3)\) 
15. \(-4x^2(y^2 + 4x + 1)\) 
16. \(-3x(3x^4 + 2x^3 - 7x^2 + 1)\) 
17. \(2x^2(3x^3 - 2x^2 + 9x - 4)\)
Use the distributive property to multiply two binomials.

\[(a + b)(c + d) = ac + ad + bc + bd\]

**Example:** Find the product \((x + 6)(x + 2)\).

\[(x + 6)(x + 2) = x(x + 2) + 6(x + 2)\]
\[= x \cdot x + x \cdot 2 + 6 \cdot x + 6 \cdot 2\]
\[= x^2 + 2x + 6x + 12\]
\[= x^2 + 8x + 12\]

**Find each product.**

1. \((x + 1)(x + 2)\)  
2. \((y + 4)(y + 5)\)  
3. \((z + 7)(z + 7)\)

4. \((y + 2)(y - 9)\)  
5. \((y + 3)(y - 3)\)  
6. \((x + 1)(x + 1)\)

7. \((x + 5)(x - 1)\)  
8. \((y + 8)(y - 3)\)  
9. \((3x + 8)(2x + 1)\)

10. \((4x + 3)(x - 6)\)  
11. \((x - 7)(3x + 4)\)  
12. \((6y + 3)(4y - 6)\)

13. \((3y - 2)(6y + 3)\)  
14. \((3x + 8)(3x - 8)\)  
15. \((2x + 4)(2x + 4)\)